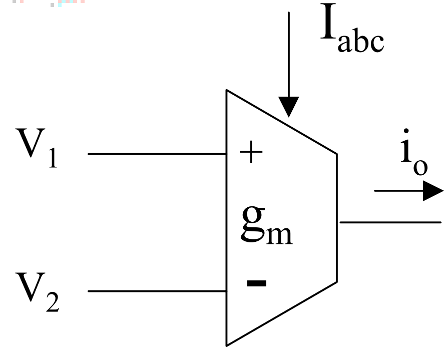
Operational Transconductance Amplifiers (OTAs)



$g_m = 19.2\, I_{abc}$

ELEN 457 (ESS)
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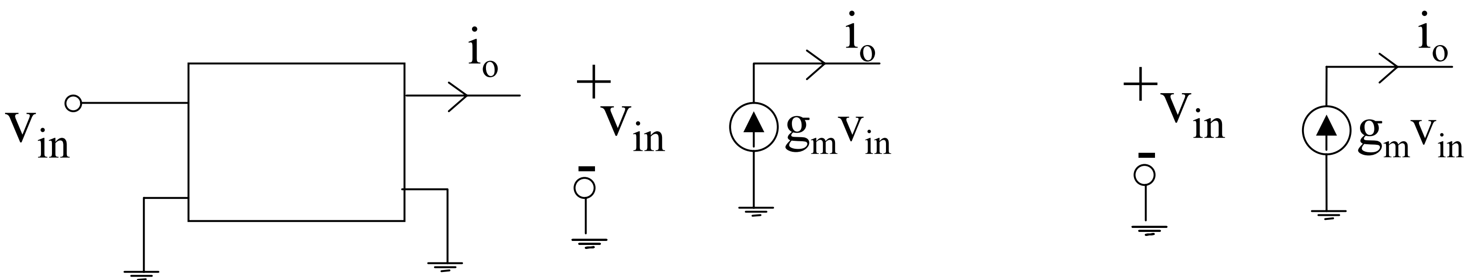
Operational Transconductance Amplifiers (OTAs)

- The input and output of an OTA are voltage and current,respectively.
- The OTA is often used in open loop. Its transconductance gain is Gm.
- The OTA input and output impedances are ideally infinite
- Integrators implemented with an Op Amp, R and C have a time constant equals to RC. There is ideally no effect of the Op Amp.
- Integrators with an OTA and a C have time constant equals to Gm/C.

- OTAs in general operate at higher frequencies than Op Amps because they have low impedance internal nodes, and operate in open loop.

See pp 620-627, section 13.3 S. Franco's textbook Mc Graw-Hill 2002 for additional material on OTAs

What is a transconductance amplifier?
It is a voltage controlled current source (VCCS)



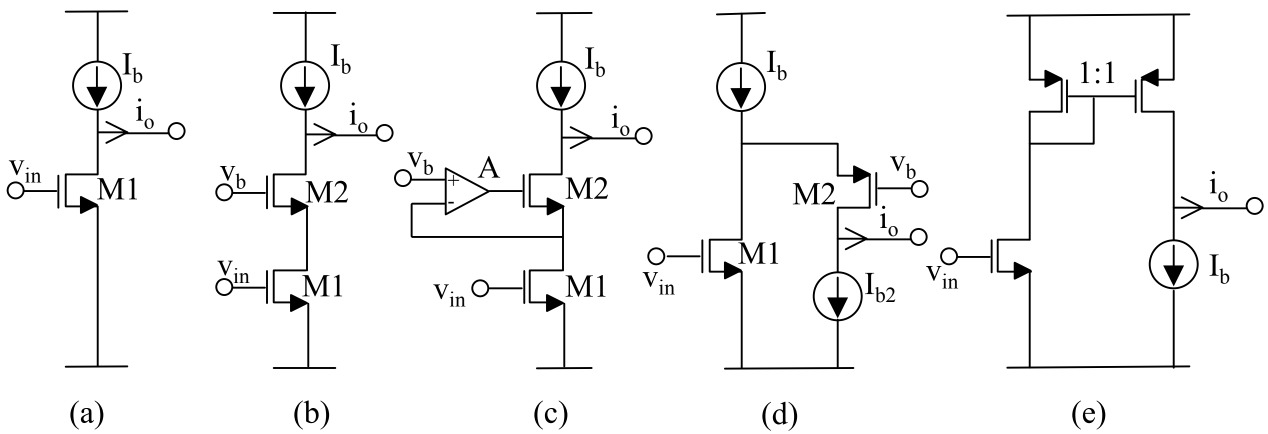
One-input/One-output
Transconductor

Non-inverting configuration
Small-signal Model

Inverting configuration
Small-signal Model

How can you implement a MOS single-input transconductor?

Single-input Transconductor Implementations



Single Input (a) Negative Simple Transconductor, (b) Cascode Transconductor, (c) Enhanced Transconductor, (d) Folded-Cascode Transconductor, (e) Positive Simple Transconductor.

- Observe that:

  $g_m = f(I_b)$, the exact relation is a function of the transistor region of operation.

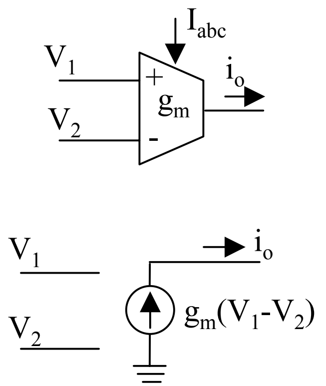
- Note that output impedance of (a) is only $1/g_{ds}$ and (b) and (c) implementations have larger output impedances.

Table 1. Properties of Simple Transconductors

Structure/ Figure	R_{out}	Min V_{DD} *
Simple/1(a)	$\frac{1}{g_{ds1}}$	$\sqrt{\frac{2I_B}{k}} + V_{sat,I_B}$
Cascode/l(b)	$\frac{g_{m2}}{g_{ds1} \quad g_{ds2}}$	$(1+m)\sqrt{\frac{2I_B}{k}} + V_{sat,I_B}$
Enhanced/1(c)	$\frac{A g_{m2}}{g_{ds1} \quad g_{ds2}}$	$(1+m)\sqrt{\frac{2I_B}{k}} + V_{sat,I_B}$
Folded/1(d)	$\frac{g_{m2}}{g_{ds1} \quad g_{ds2}}$	$\sqrt{\frac{2I_B}{k}} + V_{Tp} + V_{sat,I_B}$

* The bottom devices of the cascode pairs have an aspect ratio of $(W/L)_1/(W/L)_2=m^2$. *k* is a technological parameter determined by the mobility, and the gate oxide; $V_{sat,IB}$ is the saturation voltage for the I_B current source.

How to implement a differential input, single-ended OTA?



OTA symbol and small-signal of ideal circuit

In saturation

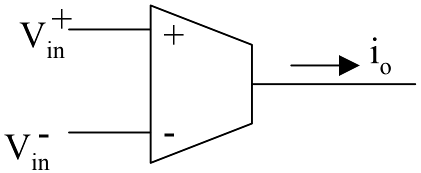
$$g_m \sim \sqrt{I_{abc}}$$

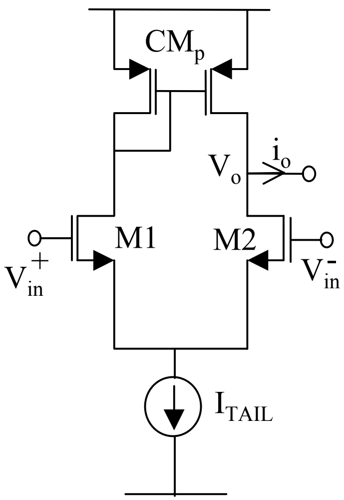
In weak inversion

$$g_m \sim I_{abc}$$

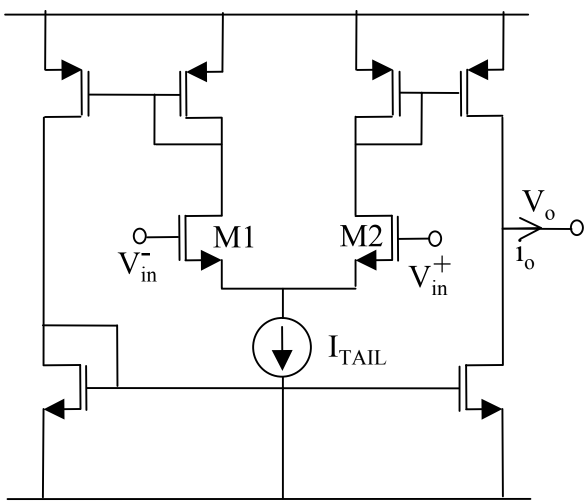
Note. The ideal OTA macromodel has infinite-input and output impedances, and g_m is frequency independent. A real OTA macromodel with non-idealities is introduced later.

Differential input OTAs



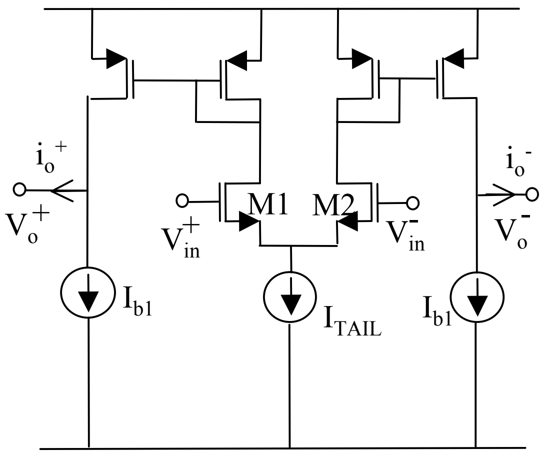


(a) Simple differential input OTA.

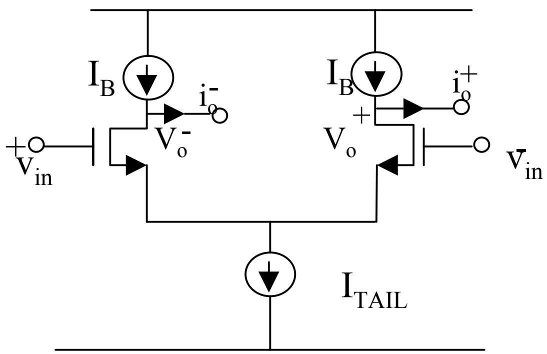


(b) Balanced OTA

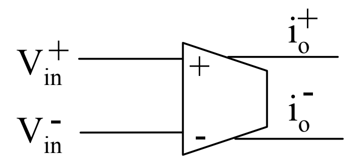
How to Implement Fully Differential Input and Output OTAs?



(c)



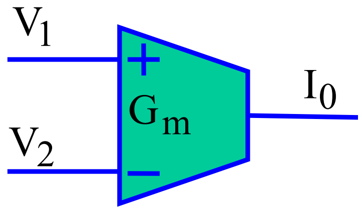
(d)



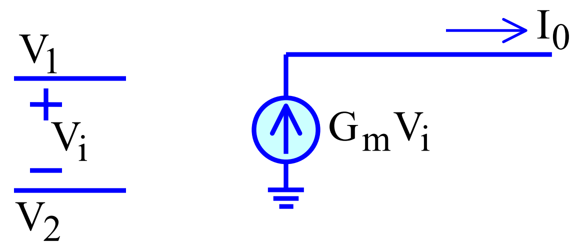
(e)

(c) Conventional fully differential OTA without CMF. (d) Simple FD OTA, suitable for HF. CMFB not shown.
(e) Fully differential OTA symbol.

OPERATIONAL TRANSCONDUCTANCE AMPLIFIER

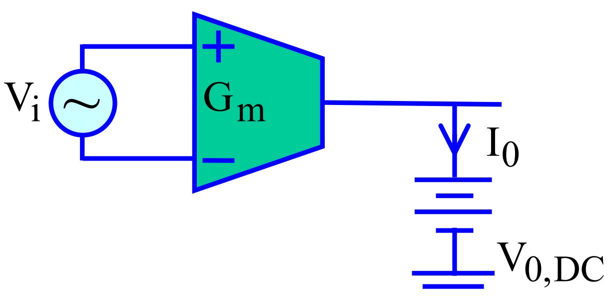


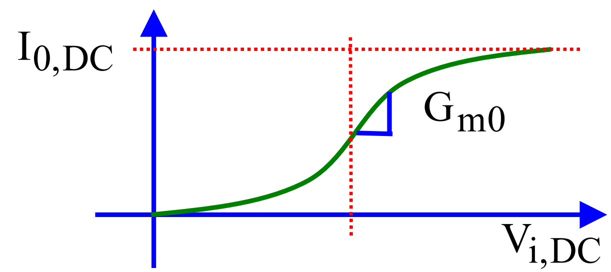
$$G_m = \left.\frac{I_0}{V_i}\right|_{V_{0(AC)}=0}$$

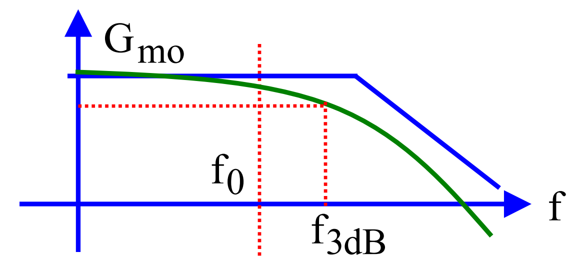


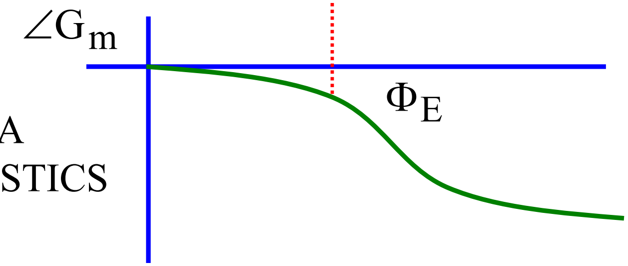
IDEAL MODEL

How do you simulate G_m?







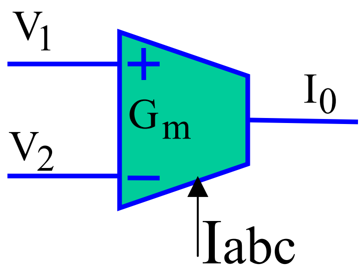


Φ_E is the phase difference between the actual G_m phase and the ideal phase.

$$\omega_p = 2\pi f_{3dB}$$

REAL OTA CHARACTERISTICS

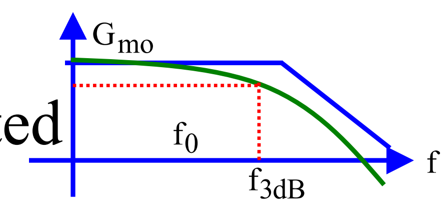
OTA real characteristics



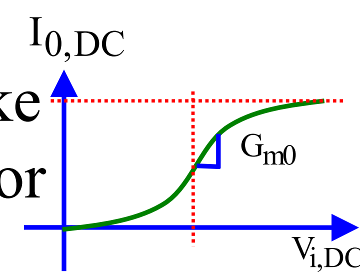
$$G_m = \left.\frac{I_0}{V_i}\right|_{V_{0(AC)=0}}$$

- $G_m = h\, I_{abc}$ for bipolar OTA or CMOS OTA operating in weak inversion. For bipolar OTA $h = q/2kT = 19.2$ for room temperature.

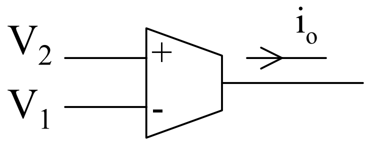
- Gm is frequency dependent and is approximated with one dominant pole



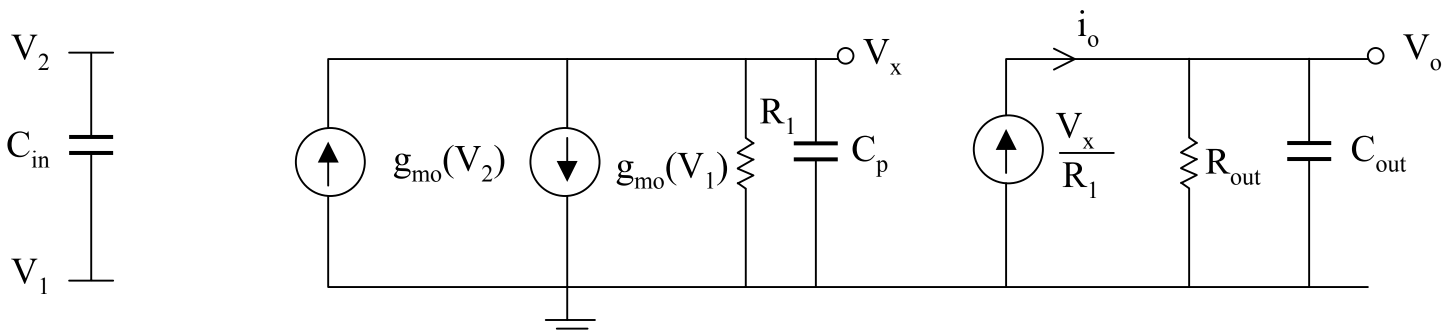
- Gm is non-linear, commercial OTA can only take less than 50mV to operate linearly. i.e. LM 1360 or RCA380



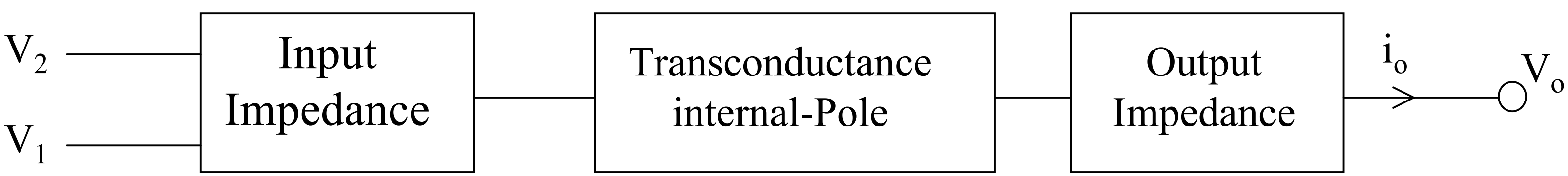
A CMOS OTA Linear Macromodel



$$g_m = \frac{g_{mo}}{1 + \dfrac{s}{\omega_p}} \quad ; \quad \omega_p = \frac{1}{R_1 C_p}$$

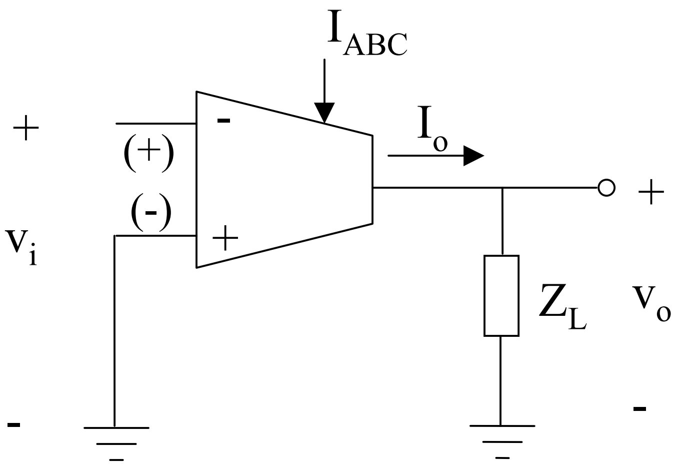


Real OTA Macromodel



OTA Macromodel Representation

Ideal OTA Basic Circuits



Bipolar OTA

$$I_o = -g_m V_i \ , \ g_m = \frac{q}{2kT} I_{ABC}$$

$$g_m = h I_{ABC}$$

$$V_o = I_o Z_L = -g_m Z_L V_i$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = -g_m Z_L$$

Non-Inverting* and Inverting Amplifier

i) Case 1 , $Z_L = R_L$ then $H(s) = -g_m R_L$ an inverting amplifier

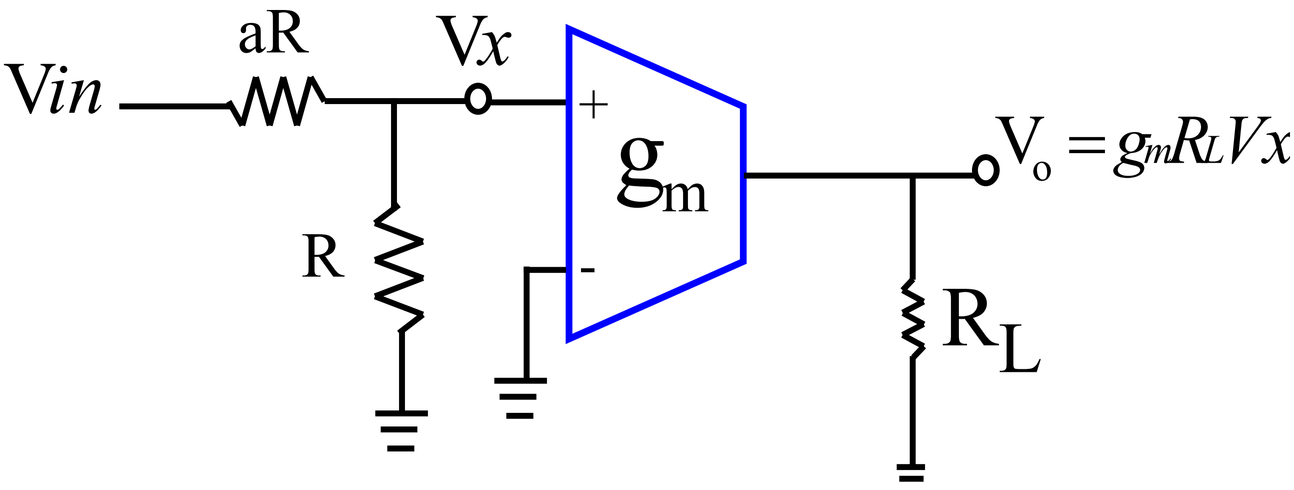
ii) Case 2 , $Z_L = 1/sC_L$ then $H(s) = -g_m/sC_L$ an integrator

iii) Case 3 , $Z_L = R_L // SC_L$ then $H(s) = \dfrac{-g_m/C_L}{s + 1/RC_L}$ First - order LP

* Note that inversion or not inversion can be accomplished simply by interchanging inputs.

How to limit the input signal to avoid burning the discrete component OTA?

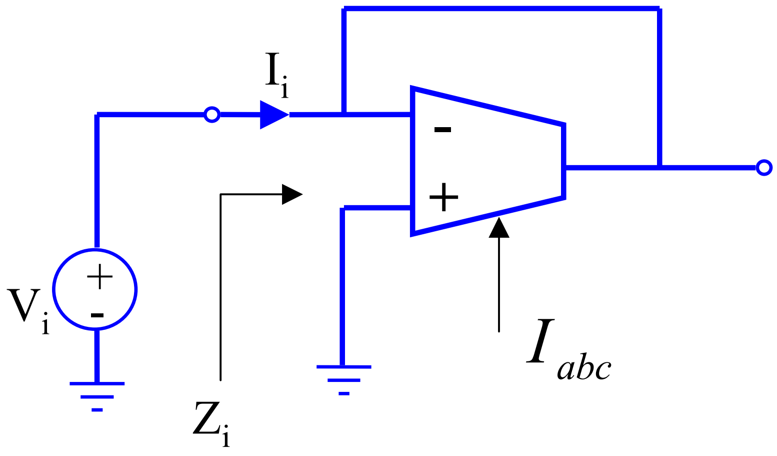
Use attenuation.

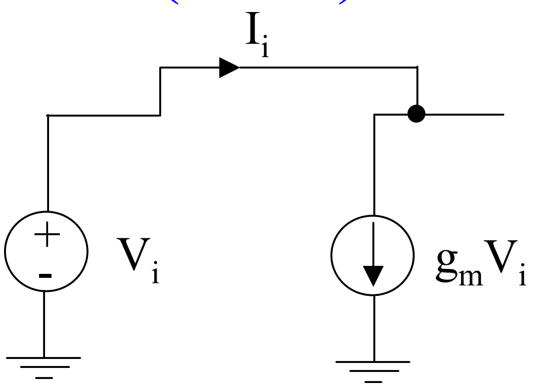


$$V_x \equiv V_{in}/(1+a)$$

Typically
a= 99, such that Vx is less than 50 mV.

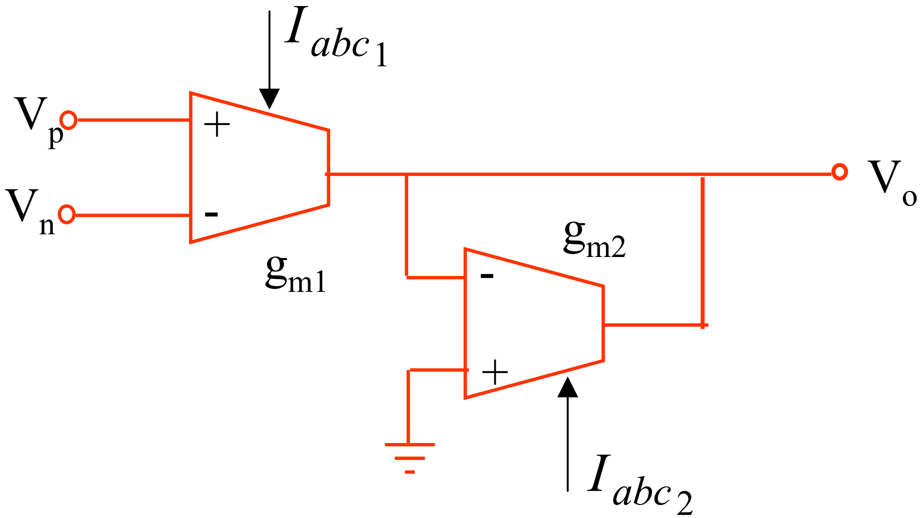
A Voltage Variable Resistor (VVR)





$$I_i = g_m V_i \Rightarrow Z_{in} = \frac{V_i}{I_i} = \frac{1}{g_m}$$

A Temperature Insensitive Amplifier

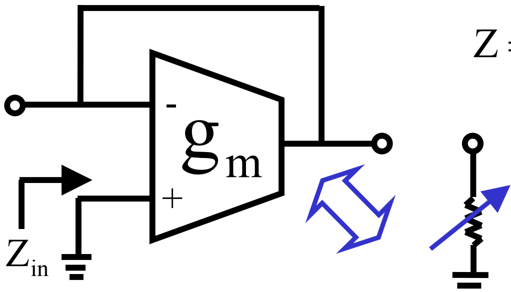


$$I_o = (V_p - V_n) g_{m1}$$

$$V_o = I_o / g_{m2}$$

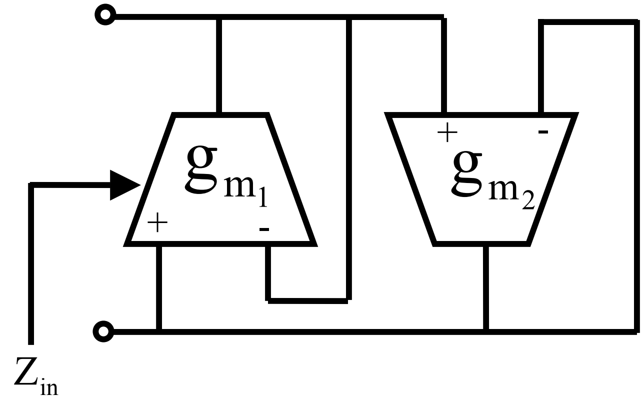
$$\frac{V_o}{V_p - V_n} = \frac{g_{m1}}{g_{m2}}$$

Controlled Impedance Elements



$$Z = \frac{1}{g_m}$$

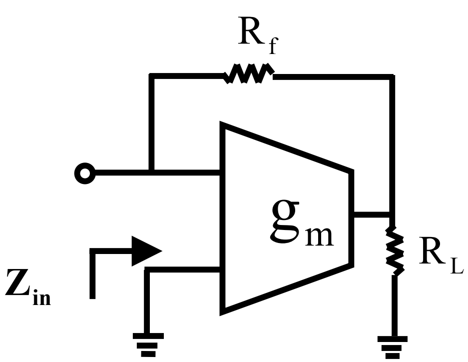
(a) Single-ended voltage variable resistor (VVR).



$$g_{m_1} = g_{m_2} = g_m$$

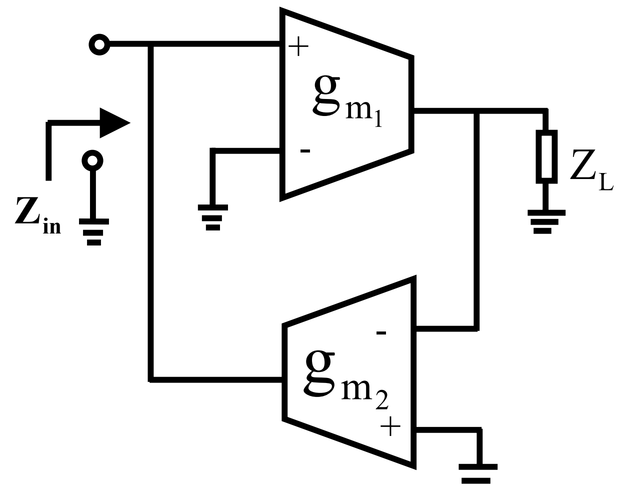
$$Z_{in} = \frac{1}{g_m}$$

(b) Floating VVR



$$Z_{in} = \frac{1 + \frac{R_f}{R_L}}{g_m}$$

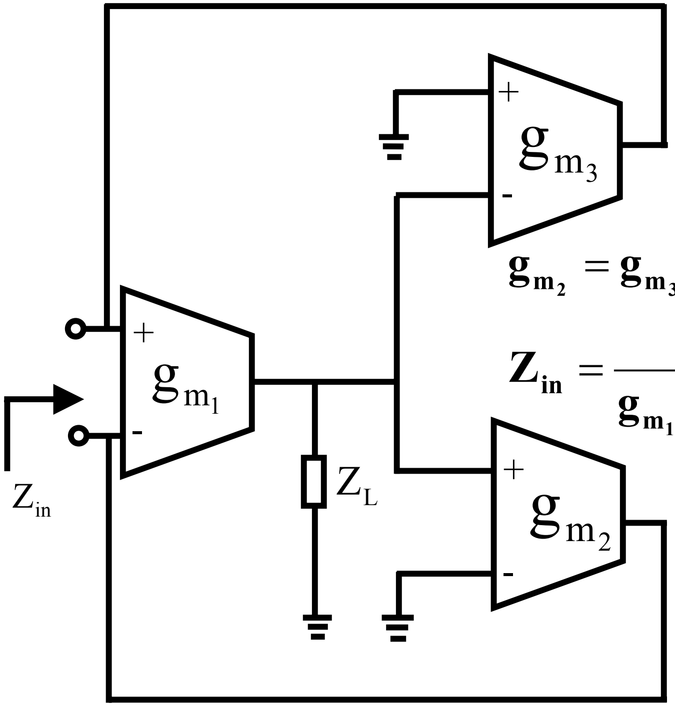
(c) Scaled VVR



$$Z_{in} = \frac{1}{g_{m_1} g_{m_2} Z_L}$$

(d) Voltage variable impedance inverter

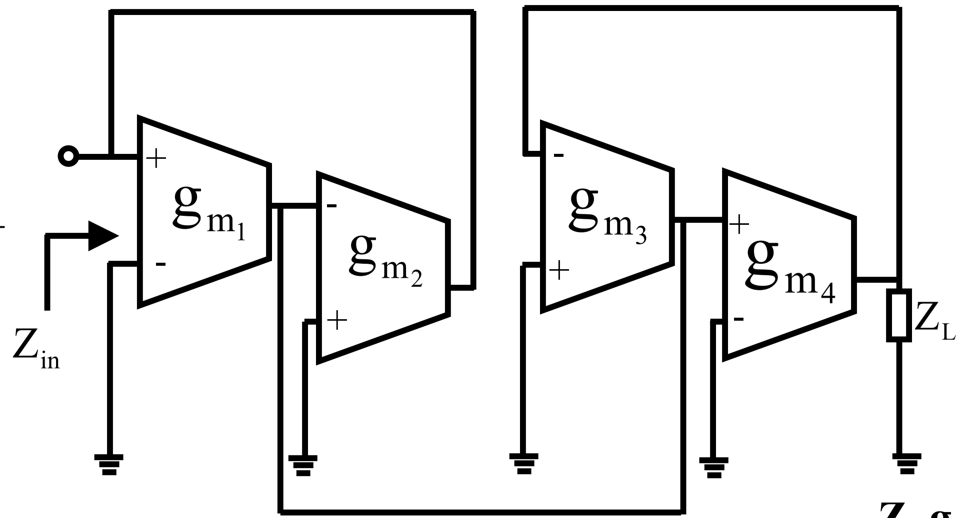
Controlled Impedance Elements



$$g_{m_2} = g_{m_3} = g_m$$

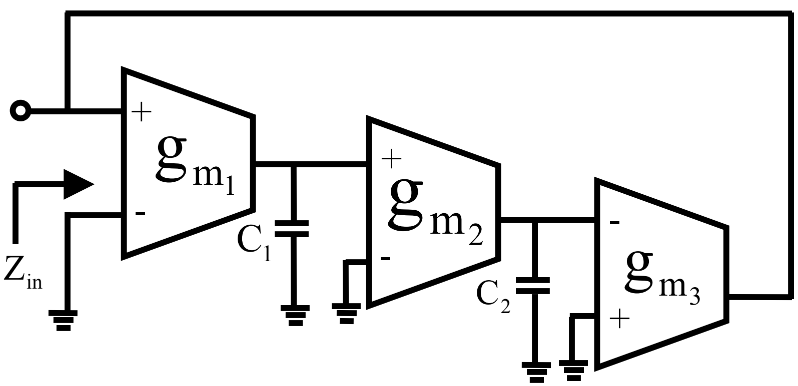
$$Z_{in} = \frac{1}{g_{m_1} g_m Z_L}$$

(e) **Voltage variable floating impedance**



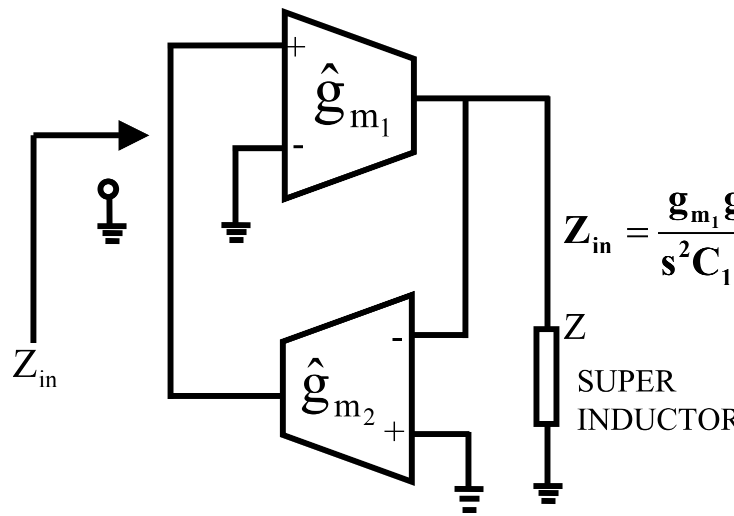
(f) Impedance multiplier

$$Z_{in} = \frac{Z_L g_{m_3} g_{m_4}}{g_{m_1} g_{m_2}}$$



$$Z_{in} = \frac{s^2 C_1 C_2}{g_{m_1} g_{m_2} g_{m_3}}$$

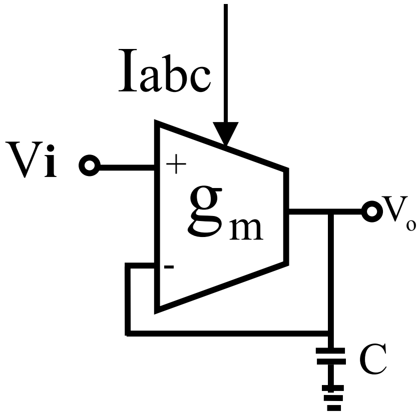
(g) Super inductor



$$Z_{in} = \frac{g_{m_1} g_{m_2} g_{m_3}}{s^2 C_1 \hat{g}_{m_1} \hat{g}_{m_2}}$$

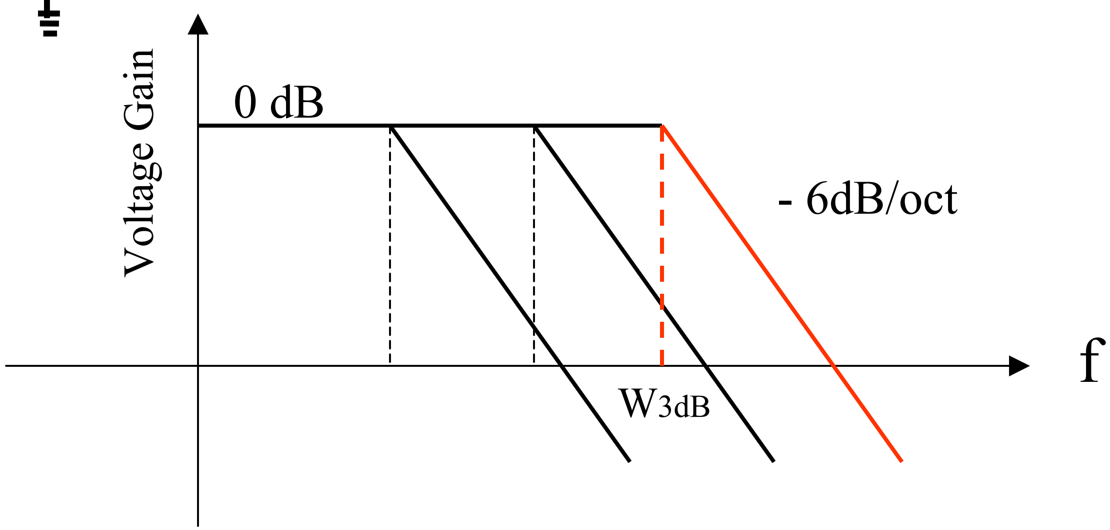
(h) FDNR

Low pass, fixed dc gain , and pole adjustable



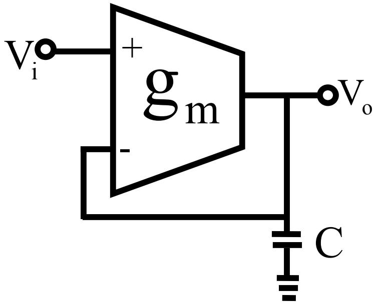
$$\frac{V_o}{V_i} = \frac{g_m}{sC + g_m}$$

$$W_{3dB} = g_m / C$$



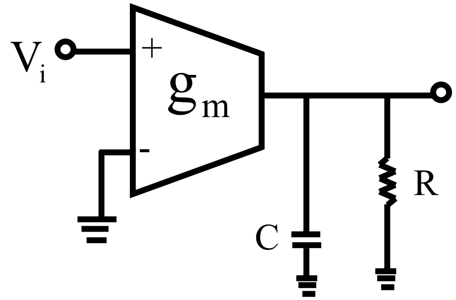
Note that we can electronically control the 3db cutoff frequency via gm, that is through Iabc

First-Order OTA-C Circuits



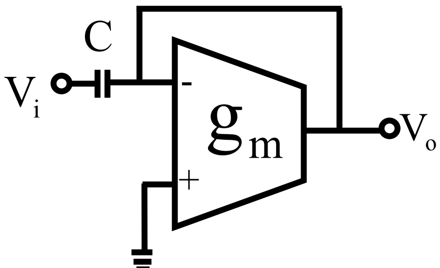
$$\frac{V_o}{V_i} = \frac{g_m}{sC + g_m}$$

(a) Lowpass, fixed dc gain pole adjustable



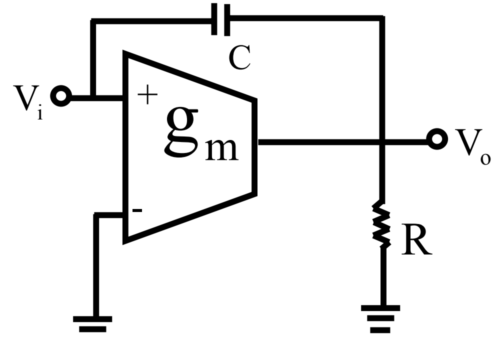
$$\frac{V_o}{V_i} = \frac{g_m}{sC + 1/R}$$

(b) Lowpass fixed pole, adjustable dc gain



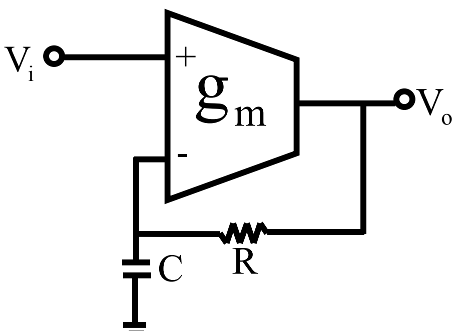
$$\frac{V_o}{V_i} = \frac{sC}{sC + g_m}$$

(c) Highpass, fixed high-frequency gain, adjustable pole



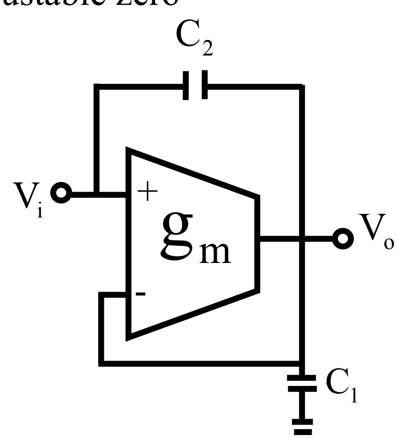
$$\frac{V_o}{V_i} = \frac{R(sC + g_m)}{sRC + 1}$$

(d) Shelving equalizer, fixed high-frequency gain, fixed pole, adjustable zero



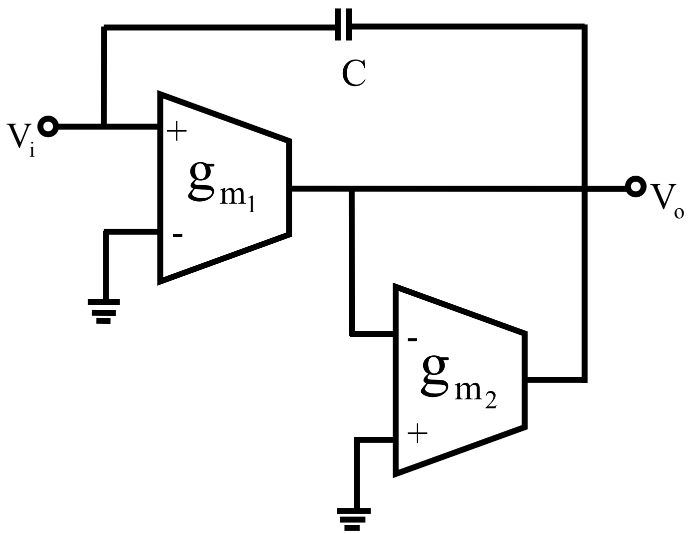
$$\frac{V_o}{V_i} = \frac{g_m(1 + RC)}{sC + g_m}$$

(e) shelving equalizer, fixed high-frequency gain, fixed zero, adjustable pole



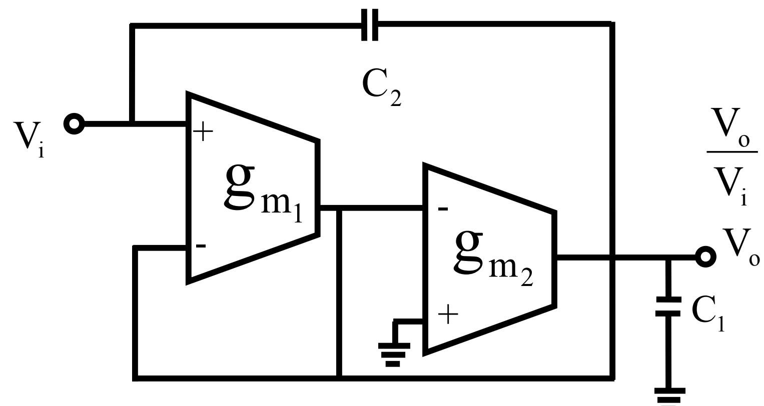
$$\frac{V_o}{V_i} = \frac{sC_2 + g_m}{s(C_1 + C_2) + g_m}$$

(f) Low Pass filter adjustable pole and zero, fixed ration



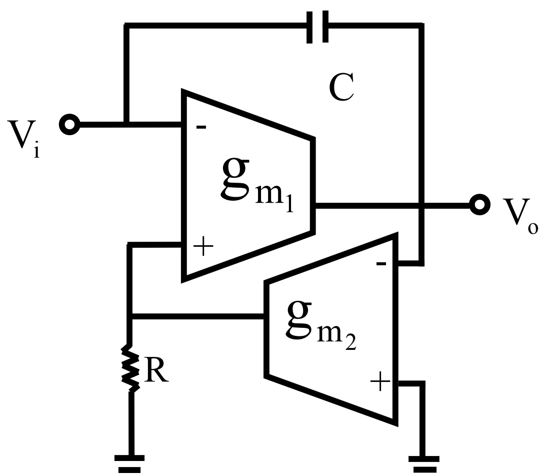
$$\frac{V_o}{V_i} = \frac{sC + g_{m_1}}{sC + g_{m_2}}$$

(g) Shelving equalizer, independently adjustable pole and zero



$$\frac{V_o}{V_i} = \frac{g_{m_1} + sC_2}{s(C_1 + C_2) + g_{m_1} + g_{m_2}}$$

(h) Low pass or highpass filter, adjustable zero and pole, fixed ratio or independent adjustment

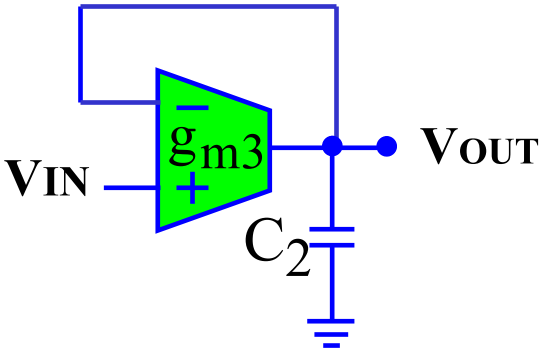


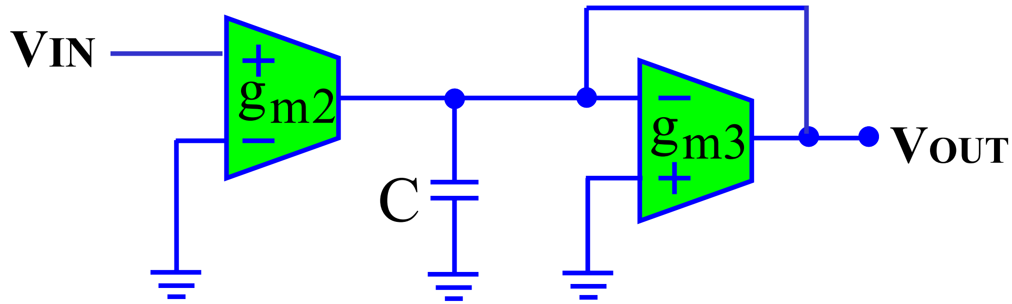
$$\frac{V_o}{V_i} = \frac{sC - g_{m_1}}{sC + g_{m_1} g_{m_2} R}$$

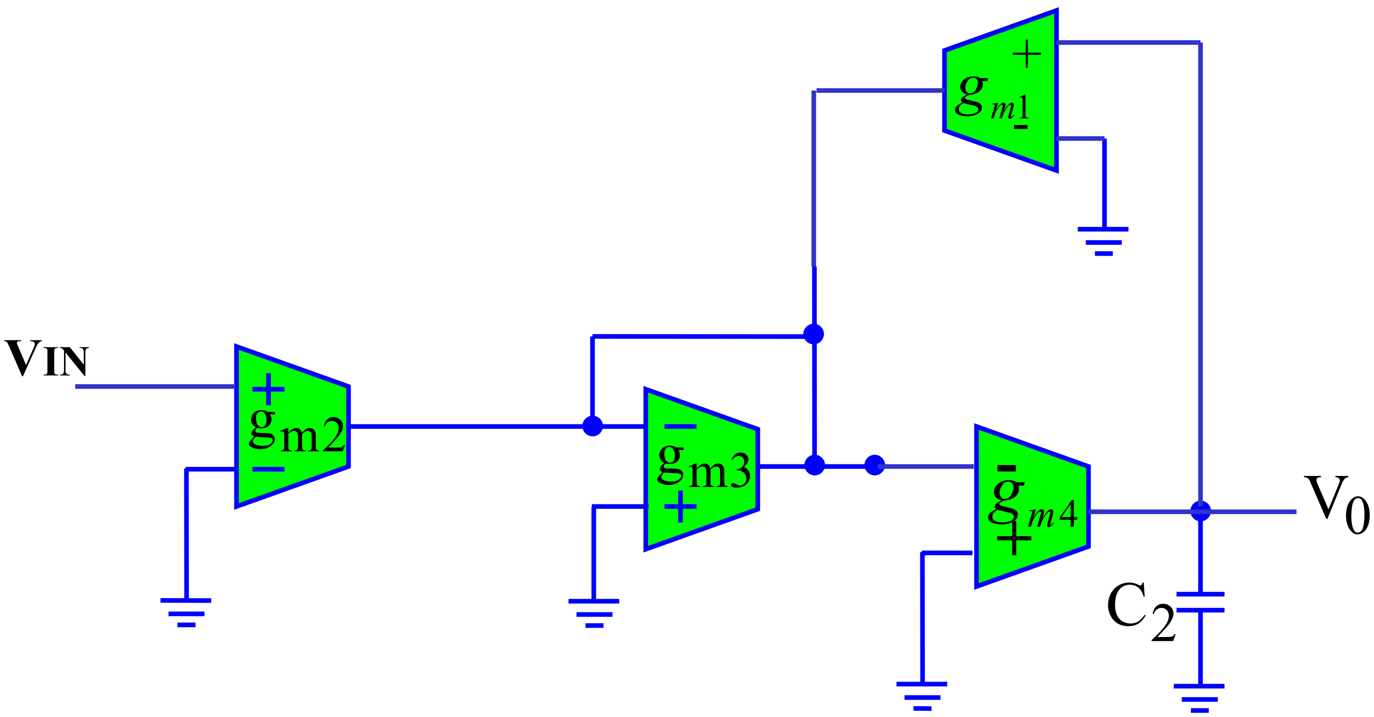
$$g_{m_2} R = 1$$

(i) Phase shifter, adjustable with g_m

LOSSY OTA-C INTEGRATORS







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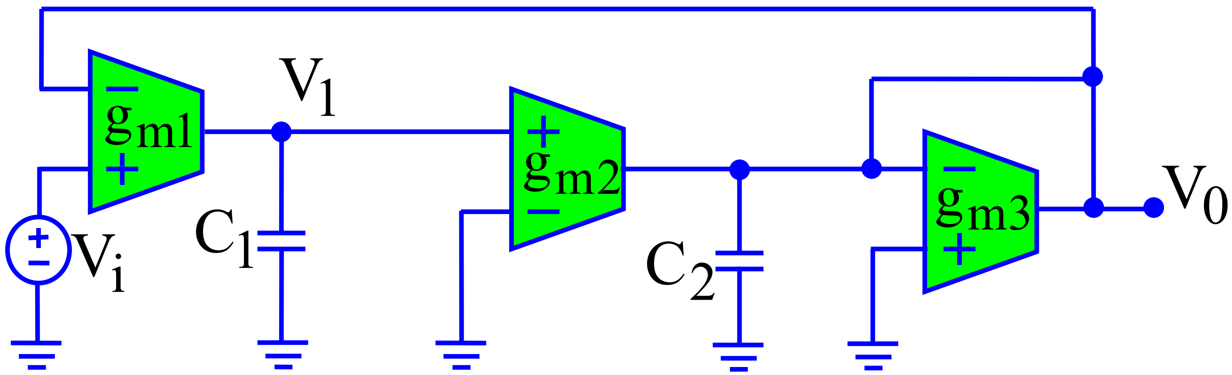
* \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
* LM13600 Dual Operational Transconductance Amplifier
* \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
*
*                Amplifier Bias Input
*                 | Diode Bias
*                 | | Positive Input
*                 | | | Negative Input
*                 | | | | Output
*                 | | | | | Negative power supply
*                 | | | | | | Buffer Input
*                 | | | | | | | Buffer Output
*                 | | | | | | | | Positive power supply
*                 | | | | | | | | |
.SUBCKT LM13600/NS  1 2 3 4 5 6 7 8 11
 * Features:
* gm adjustable over 6 decades.
* Excellent gm linearity.
* Linearizing diodes.
* Controlled impedance buffers.
* Wide supply range of +/-2V to +/-22V.
*
* Note:  This model is single-pole in nature and over-estimates
*       AC bandwidth and phase margin (stability) by over 2X.
*       Although refinement may be possible in the future, please
*       use benchtesting to finalize AC circuit design.
*
* Note: Model is for single device only and simulated
*       supply current is 1/2 of total device current.
*
************************************************************
*
C1  6  4  4.8P
C2  3  6  4.8P
* Output capacitor
C3  5  6  6.26P
D1  2  4  DX
D2  2  3  DX
D3  11 21 DX
D4  21 22 DX
```

```
D5  1  26 DX
D6  26 27 DX
D7  5  29 DX
D8  28 5  DX
D10 31 25 DX
*  Clamp for -CMR
D11 28 25 DX
* Ios source
F1  4  3  POLY(1)   V6 1E-10 5.129E-2 -1.189E4 1.123E9
F2  11 5  V2        1.022
F3  25 6  V3        1.0
F4  5  6  V1        1.022
F5  30 6  V3        1.0
* Output impedance
F6  5  0  POLY(2)   V3 V7 0 0 0 0 1
G1  0  33 5         0 .55E-3
I1  11 6  300U
Q1  24 32 31        QX1
Q2  23 3  31        QX2
Q3  11 7  30        QX1
Q4  11 30 8         QY
V1  22 24 0V
V2  22 23 0V
V3  27 6  0V
V4  11 29 1.4
V5  28 6  1.2
V6  4  32 0V
V7  33 0  0V
.MODEL QX1 NPN (IS=5E-16     BF=200 NE=1.15 ISE=.63E-16 IKF=1E-2)
.MODEL QX2 NPN (IS=5.125E-16 BF=200 NE=1.15 ISE=.63E-16 IKF=1E-2)
.MODEL QY  NPN (IS=6E-15     BF=140)
.MODEL DX  D   (IS=5E-16)
.ENDS
*$
```

OTA-C Three OTA Filter: Transfer Function Derivation



Assume ideal OTAs first, then :

$$V_1 = \frac{1}{sC_1} g_{m1}(V_i - V_0) \qquad (1)$$

$$V_0 = (g_{m2}V_1 - g_{m3}V_0)\frac{1}{sC_2} \qquad (2)$$

(1) into (2)

$$(sC_2)V_0 = \left[g_{m2} \frac{g_{m1}}{sC_1}(V_i - V_0) - g_{m3}V_0 \right]$$

$$V_0\left[sC_2 + \frac{g_{m1}g_{m2}}{sC_1} + g_{m3}\right] = \frac{g_{m1}g_{m2}}{sC_1}V_i$$

$$H_{LP}(s) = \frac{V_0}{V_i} = \frac{g_{m1}g_{m2}}{s^2C_1C_2 + sC_1g_{m3} + g_{m1}g_{m2}} = \frac{\frac{g_{m1}g_{m2}}{C_1C_2}}{s^2 + s\frac{g_{m3}}{C_2} + \frac{g_{m1}g_{m2}}{C_1C_2}}$$

$$\omega_o^2 = \frac{g_{m1}g_{m2}}{C_1C_2} \quad , \quad BW = \frac{\omega_o}{Q} = \frac{g_{m3}}{C_2}$$

$$Q = \frac{1}{g_{m3}}\sqrt{\frac{g_{m1}g_{m2}C_2}{C_1}} = \frac{C_2\omega_o}{g_{m3}}$$

Now let's assume the transconductance is characterized by:

$$g_m = g_{mo} e^{-s/\omega_p} \cong g_{mo}\left(1 - s/\omega_p\right) \text{ for } \omega_p << \omega_o.$$

Under this condition the excess phase can be expressed as $\phi_E \cong \omega_o / \omega_p$.

Note that ideally $\phi_E = 0^0$.

then,

$$H_{LP}(s) = \frac{g_{mo1} g_{mo2} (1 - s/\omega_{p1})(1 - s/\omega_{p2})}{s^2 C_1 C_2 + s C_1 g_{mo3} (1 - s/\omega_{p3}) + g_{mo1} g_{mo2} (1 - s/\omega_{p1})(1 - s/\omega_{p2})}$$

$$D(s) = s^2 C_1 C_2 + s C_1 g_{mo3} - s^2 \frac{C_1 g_{mo3}}{\omega_{p3}} + g_{mo1} g_{mo2} \left(1 - s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}}\right)$$

$$D(s) = s^2 \left\{C_1 C_2 - \frac{C_1 g_{mo3}}{\omega_{p3}} + \frac{g_{mo1} g_{mo2}}{\omega_{p1}\omega_{p2}}\right\} + s\left\{C_1 g_{mo3} - \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) g_{mo1} g_{mo2}\right\} + g_{mo1} g_{mo2}$$

Then the actual ω_{oa} and BW_a become

$$\omega_{oa}^2 = \frac{g_{mo1} g_{mo2}}{C_1 C_2 + \frac{g_{mo1} g_{mo2}}{\omega_{p1}\omega_{p2}} - \frac{C_1 g_{mo3}}{\omega_{p3}}}$$

$$BW_a = \frac{\omega_{oa}}{Q_a} = \frac{C_1 g_{mo3} - \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) g_{mo1} g_{mo2}}{C_1 C_2 - \frac{C_1 g_{mo3}}{\omega_{p3}} + \frac{1}{\omega_{p1}\omega_{p2}}}$$

Let us also assume that $\omega_{p1} = \omega_{p2} = \omega_p$, then $\omega_{oa} \cong \omega_o$, thus,

$$BW_a = \frac{\omega_{oa}}{Q_a} = \frac{C_1 g_{mo3} - \frac{2}{\omega_{p1}} g_{mo1} g_{mo2}}{C_1 C_2 - \frac{C_1 g_{mo3}}{\omega_p} + \frac{1}{\omega_p^2}} \cong \frac{C_1 g_{mo3} - \frac{2}{\omega_p} g_{mo1} g_{mo2}}{C_1 C_2}$$

$$BW_a \cong \frac{g_{mo3}}{C_2} - \frac{g_{mo1} g_{mo2}}{C_1 C_2} \frac{2}{\omega_{p1}} = BW - \omega_{oa}^2 \cdot \frac{2}{\omega_{p1}} = BW - \frac{2\omega_{oa}^2}{\omega_{p1}}$$

$$Q_a = \frac{\omega_{oa}}{BW_a} \cong \frac{1 \cdot \omega_{oa}}{\frac{g_{mo3}}{C_2} - \omega_{oa}^2 \frac{2}{\omega_{p1}}}$$

$$Q_a = \frac{\frac{C_2 \omega_{oa}}{g_{m3}}}{1 - \frac{C_2 \omega_{oa}}{g_{m3}} \frac{2\omega_{oa}}{\omega_{p1}}} = \frac{Q}{1 - \frac{Q 2\omega_{oa}}{\omega_{p1}}} = \frac{Q}{1 - \frac{2\omega_{oa}}{\omega_{p1}} Q}$$

Alternatively, Q_a can be expressed in terms of the excess phase $\phi_E = \tan^{-1} \frac{\omega_o}{\omega_p} \cong \frac{\omega_o}{\omega_p}$ then

$$Q_a \cong \frac{Q}{1 - 2\phi_E Q} \cong Q(1 + 2\phi_E Q)$$

$$BW_a = BW - 2\omega_{oa}\phi_E$$

Furthermore, if $A_{vo} = g_m R_o$ is taken into account, then

$$Q_a = \frac{Q}{1 + \frac{2Q}{A_{vo}}}$$

If $A_{vo} = 500$

$$Q_a = \frac{Q}{1 + 4 \times 10^{-3} Q}$$

Q	Q_a
1	0.996
5	4.902
10	9.6
50	41.667

Note that :

$Q_a \downarrow$ when $A_{vo} \downarrow$

$BW_a \downarrow$ $Q_a \uparrow$ when $\phi_E \uparrow$

$$Q_a \cong \frac{Q}{1 - 2\phi_E Q} \cong Q(1 + 2\phi_E Q)$$

$$BW_a = BW - 2\omega_{oa}\phi_E$$

References

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[4] E.Sánchez-Sinencio and J. Silva-Martinez, “ CMOS Transconductance Amplifiers, Architectures and Active Filters: A Tutorial, *IEE Proceedings, Circuits Devices and System,”*