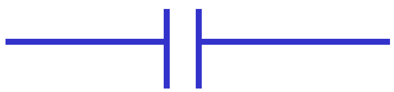
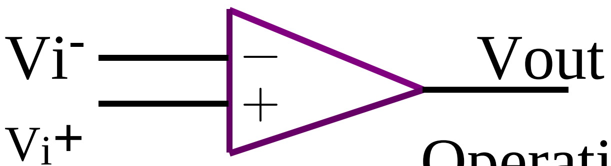
Building Blocks for Active-RC and OTA-C Filters

Building Blocks for Active-RC and OTA-C Filters

Resistors R

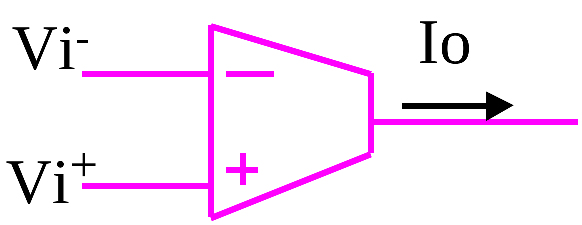


Capacitors C



VCVS

Operational Amplifier (Op Amp)

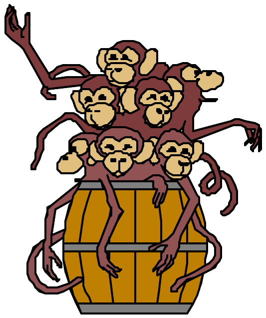


VCCS

Operational Transconductance Amplifier (OTA)

COMPONENTS

When do we use an OTA or an Op Amp ?



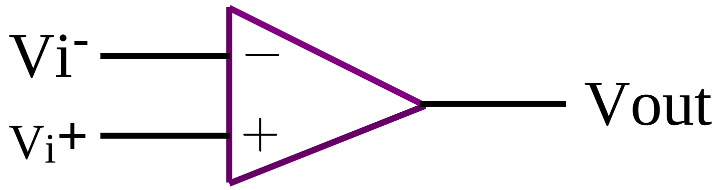
• An Operational Transconductance Amplifier is a voltage control current source VCCS. It has ideally an infinite bandwidth, input impedance and output impedance.

• An OTA can be used for open-loop continuous-time applications or for closed-loop applications when the load impedance is high, i.e., OTA-C or Switched-Capacitor Circuits.

• An Operational Amplifier is a VCVS has ideally infinite gain-bandwidth product and input impedance and zero output impedance.

• Op Amps are used in closed loops unless they are used as comparators. They can handle low impedance loads such as speakers

Operational Amplifier



$Av = Vout/(Vi^{+} - Vi^{-})$

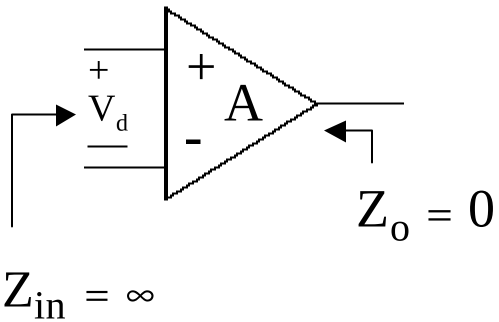
Ideal characteristics:

- **Infinite voltage gain and GBW**
- **Zero output impedance**
- **Infinite Input Impedance**
- **Infinite common-mode rejection ratio**

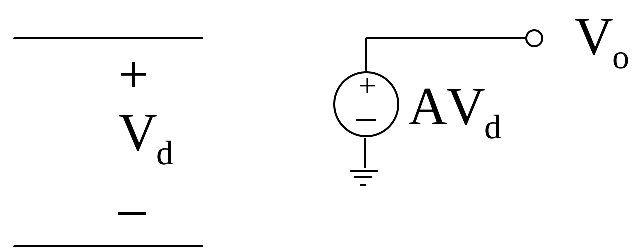
Always use the Op Amp in closed loop for amplification

Basic Building Blocks

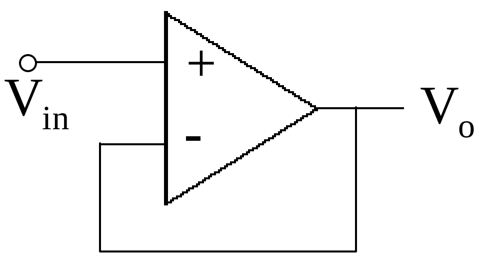
- The Op Amp

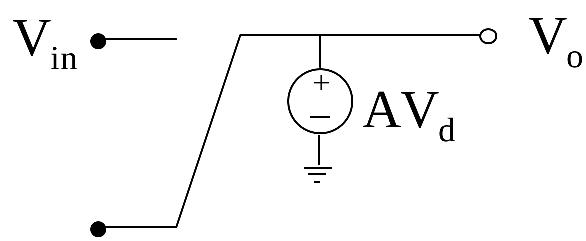


- Ideal Case $A \rightarrow \infty$



- BUFFER

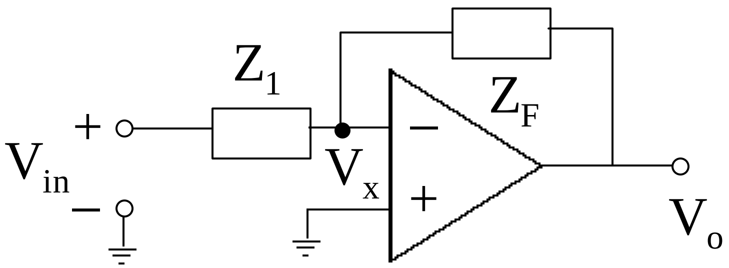


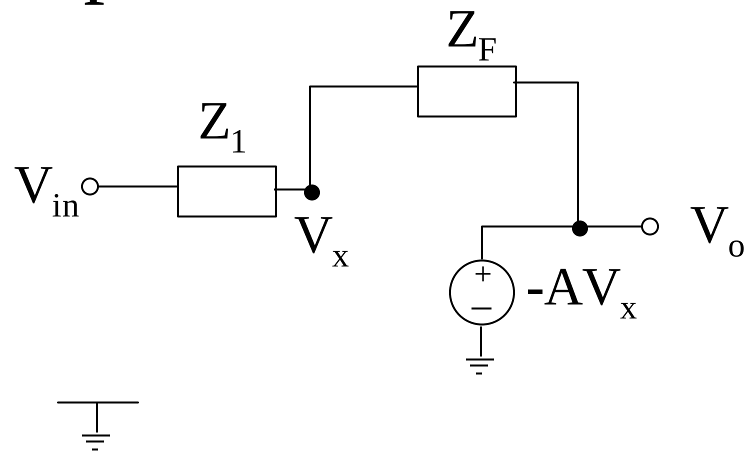


$$V_o = AV_d = A(V_{in} - V_o)$$

$$V_o = \frac{V_{in} A}{1 + A} = \left.\frac{V_{in}}{\frac{1}{A} + 1}\right|_{A = \infty} = V_{in}$$

Fundamental Operators





INVERTER

$$V_x\left(\frac{1}{Z_1}+\frac{1}{Z_F}\right)-\frac{V_o}{Z_F}=\frac{V_{in}}{Z_1} \qquad (1)$$

$$V_o=-AV_x \qquad (2)$$

From (2)

$$V_X=\frac{-V_o}{A}$$

$$\frac{V_o}{V_{in}}=-\frac{\frac{Z_F}{Z_1}}{1+\frac{1}{A}\left(1+\frac{Z_F}{Z_1}\right)}$$

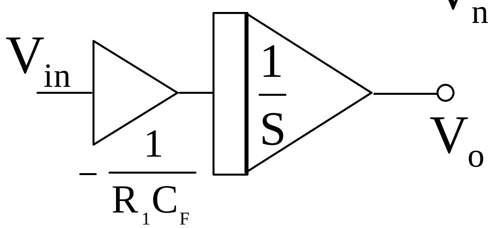
$$\left.\frac{V_o}{V_{in}}\right|_{A\to\infty}=-\frac{Z_F}{Z_1}$$

$$\left.V_x\right|_{A\to\infty}=0$$

Particular Cases

- If $Z_1 = R_1$, $Z_F = R_F$

$$\frac{V_o}{V_{in}} = -\frac{R_F}{R_1}$$



- Integrator

$$Z_1 = R_1, Z_F = \frac{1}{SC_F}$$

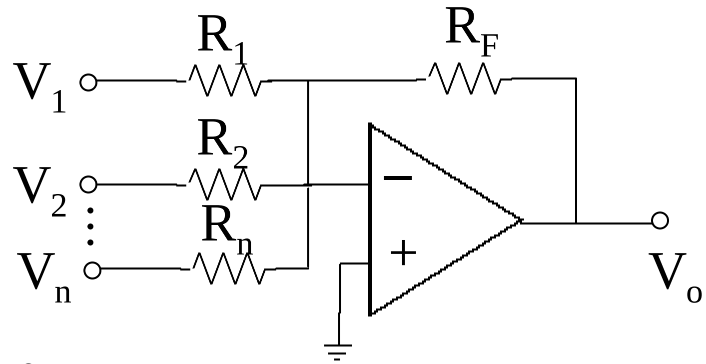
$$\frac{V_o}{V_{in}} = -\frac{1}{SR_1C_F}$$

- Differentiator

$$Z_1 = \frac{1}{SC_1}, Z_F = R_F$$

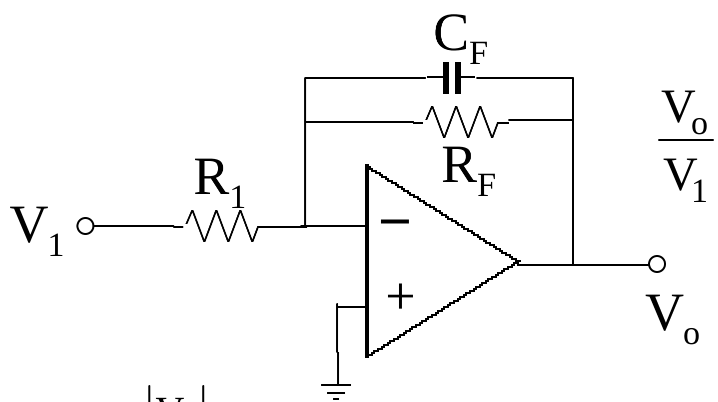
$$\frac{V_o}{V_{in}} = -SC_1R_F$$

- Multiple Inputs

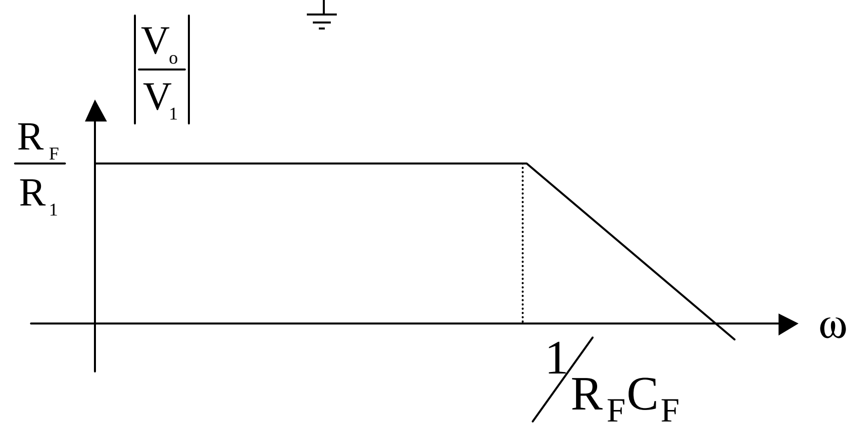


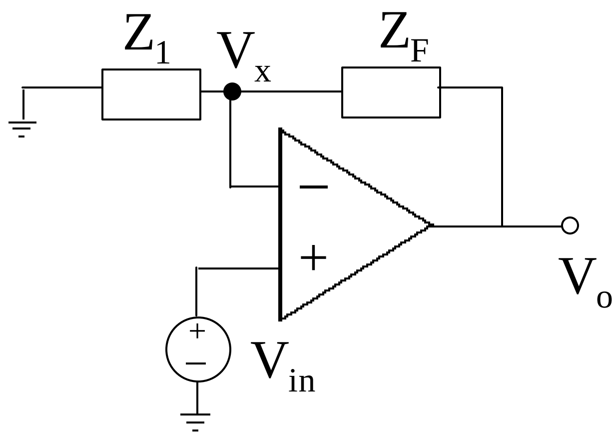
$$V_o = -\sum_{i=1}^{n} \frac{R_F}{R_i} V_i$$

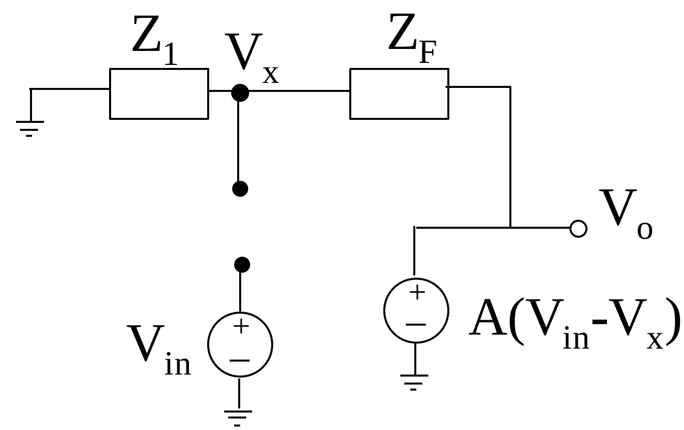
- Low Pass (First-Order)



$$\frac{V_o}{V_1} = -\frac{Z_F}{Z_1} = \frac{\frac{-R_F}{R_1}}{1 + SC_FR_F}$$







NON-INVERTER

EQUIVALENT CIRCUIT

$$V_x\left(\frac{1}{Z_1}+\frac{1}{Z_F}\right)-\frac{V_o}{Z_F}=0$$

$$V_o = A(V_{in} - V_x)$$

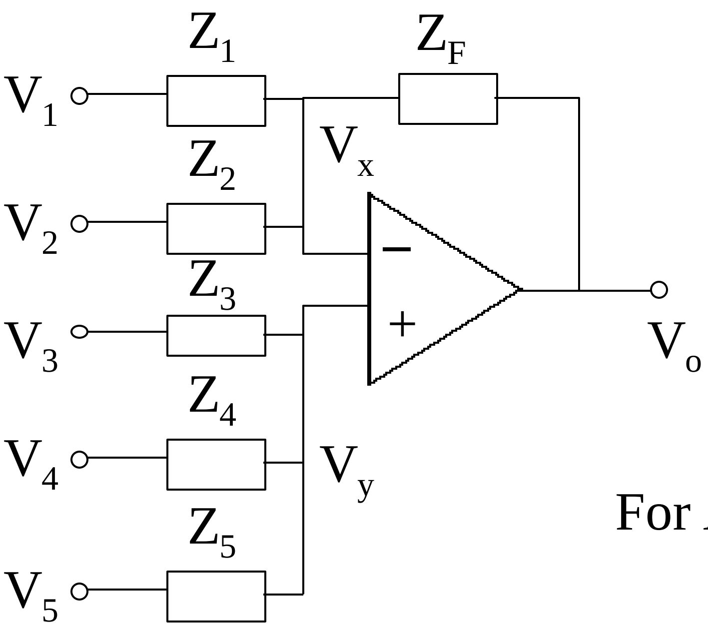
$$\frac{V_o}{V_{in}} = \frac{1+\frac{Z_F}{Z_1}}{1+\left(1+\frac{Z_F}{Z_1}\right)\frac{1}{A}}\Bigg|_{A\to\infty} = 1+\frac{Z_F}{Z_1}$$

- Particular Cases

$Z_1 = R_i$, $Z_F = R_F$
Non-Inverter Amp

$$\frac{V_o}{V_{in}} = 1+\frac{R_F}{R_1}$$

How to Make a General Summer?



$$V_x\left(\frac{1}{Z_F}+\frac{1}{Z_1}+\frac{1}{Z_2}\right)-\frac{V_1}{Z_1}-\frac{V_2}{Z_2}-\frac{V_o}{Z_F}=0 \qquad (1)$$

$$V_y\left(\frac{1}{Z_3}+\frac{1}{Z_4}+\frac{1}{Z_5}\right)-\frac{V_3}{Z_3}-\frac{V_4}{Z_4}-\frac{V_5}{Z_5}=0 \qquad (2)$$

$$V_o = A(-V_x + V_y) \qquad (3)$$

For $A \to \infty$ and from (3) $\quad V_y = V_x$

Now if we let $Z_2=Z_3$, $Z_1=Z_4$ and $Z_5=Z_F$, one can equate (1) & (2)

$$-\frac{V_1}{Z_1}-\frac{V_2}{Z_2}-\frac{V_o}{Z_F}=-\frac{V_3}{Z_3}-\frac{V_4}{Z_4}-\frac{V_5}{Z_5}$$

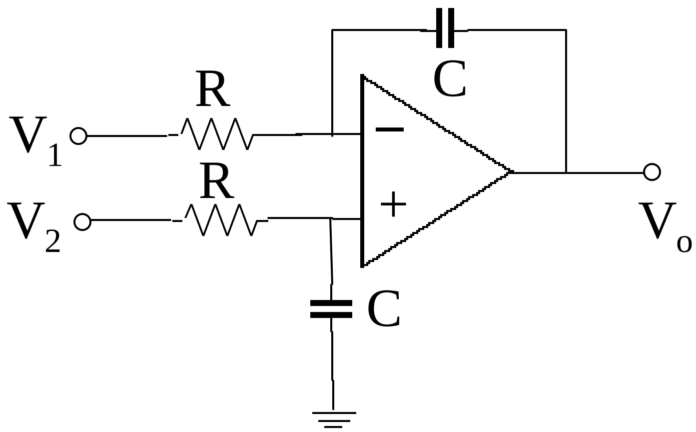
$$V_o=-\frac{Z_F}{Z_1}V_1-\frac{Z_F}{Z_2}V_2+\frac{Z_F}{Z_3}V_3+\frac{Z_F}{Z_4}V_4+\frac{Z_F}{Z_5}V_5$$

Reader: What are, in general, the conditions for the above equation to hold?

General Summer

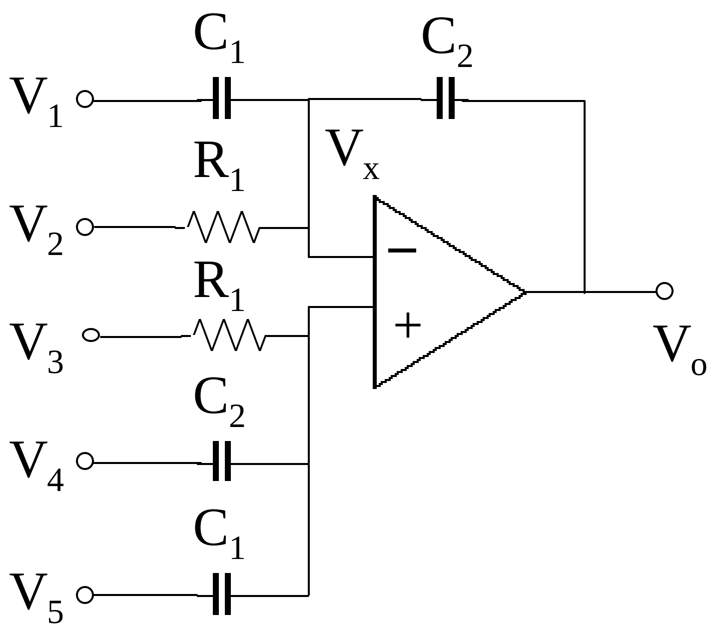
(Applications)

Integrator



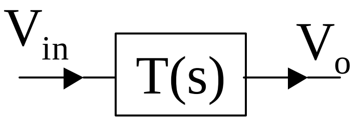
$$V_o(s) = -\frac{V_1(s)}{sRC} + \frac{V_2(s)}{sRC}$$

Mixed-Operations



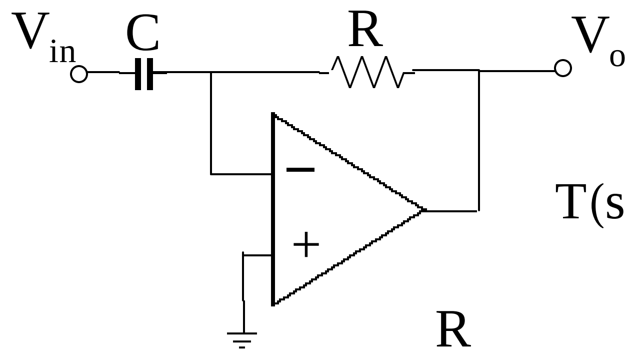
$$V_o(s) = -\frac{C_1}{C_2}V_1(s) - \frac{V_2(s)}{sR_1C_2} + \frac{V_3(s)}{sR_1C_2} + V_4(s) + \frac{C_1}{C_2}V_5(s)$$

Additional **Feedback Properties**



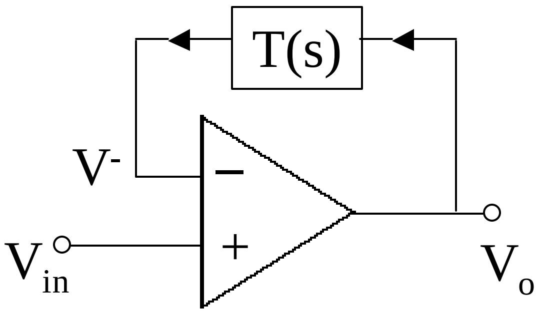
$$T(s) = \frac{V_o(s)}{V_{in}(s)}$$

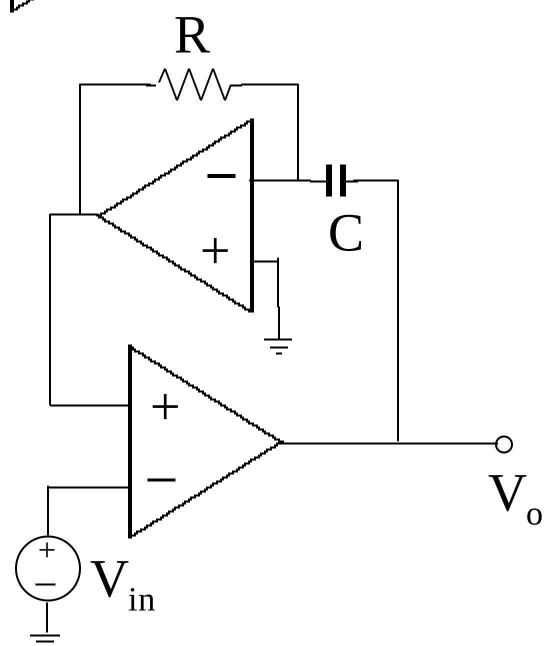
Example



$$T(s) = -SRC$$

If T(s) becomes part of the negative feedback:





What is $H(s) = \frac{V_o(s)}{V_{in}(s)}$?

Due to the ideal Op Amp

$V_{in} = V^-$ and $T(s) = \frac{V^-(s)}{V_o(s)} = \frac{V_{in}(s)}{V_o(s)} = \frac{1}{H(s)}$

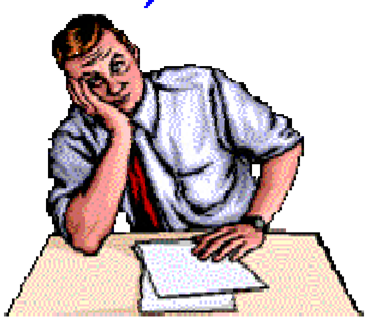
Thus $H(s) = \frac{1}{T(s)}$

$$H(s) = \frac{1}{T(s)} = -\frac{1}{SRC}$$

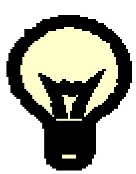
What is the common-mode gain voltage ?

What is the differential gain voltage ?

What is the common-mode rejection ratio (CMRR)?

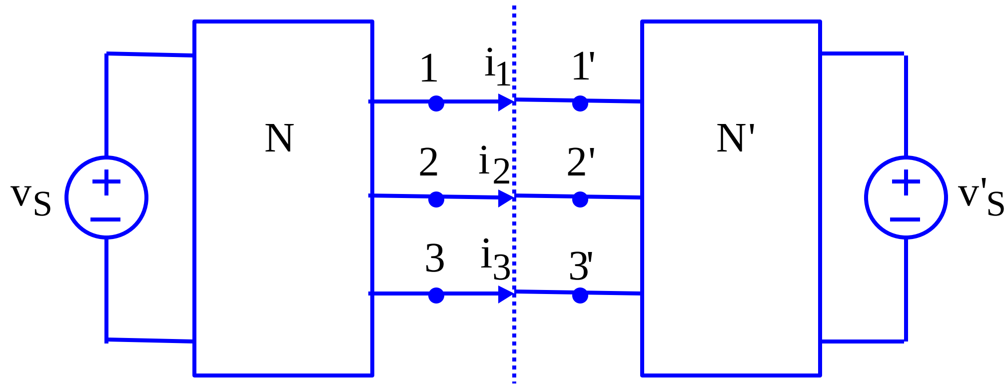


Do we need a fully differential op amp to have a good CMRR ?

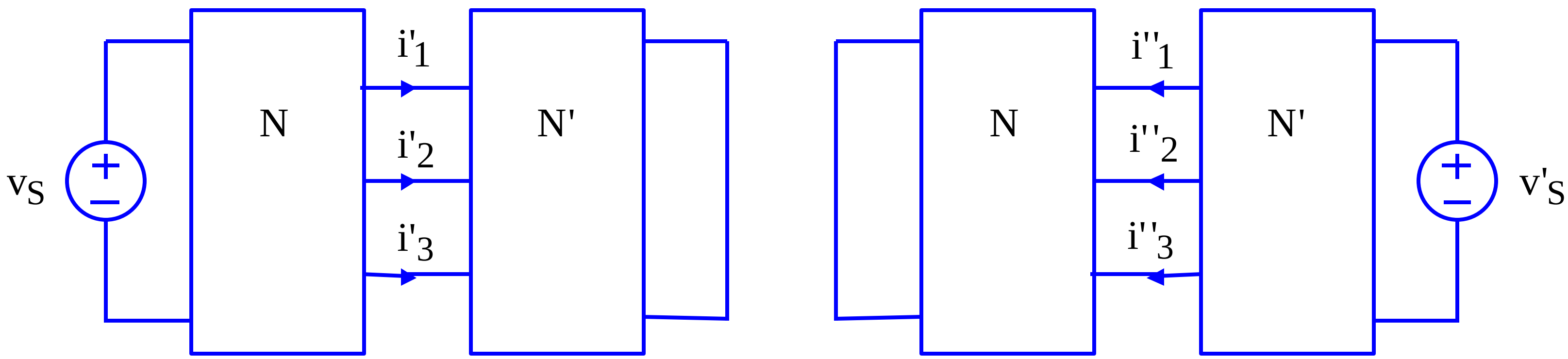


How about using single ended op amps yielding a fully differential Output ?

Symmetrical Circuits



Symmetrical Circuit



Application of Superposition

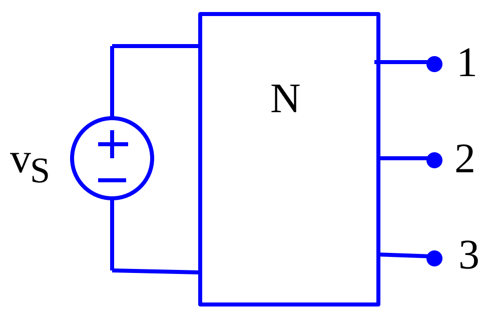
$$\left.\begin{aligned} i_1 &= i'_1 + i''_1 \\ i_2 &= i'_2 + i''_2 \\ i_3 &= i'_3 + i''_3 \end{aligned}\right\}$$

Now since $v_S = v'_S$

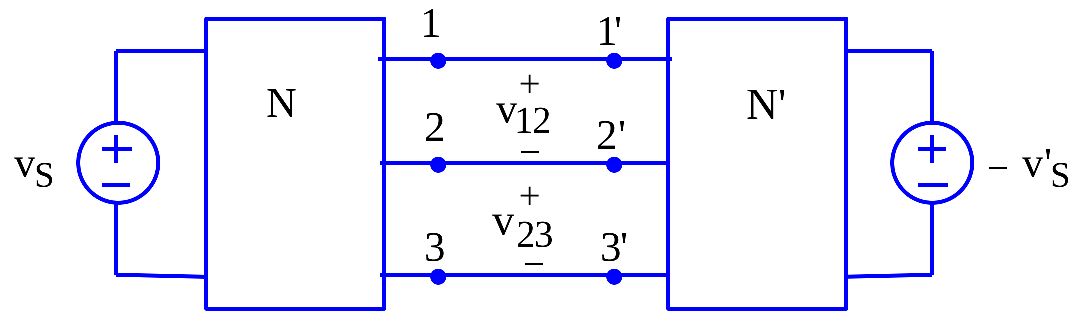
$i'_j = -i''_j \quad , \quad j = 1,2,3$

so $i_j = 0 \quad , \quad j = 1,2,3$

So we can simplify the analysis to half circuit for equal (common) inputs.



Symmetrical Input (common-mode)



For antisymmetrical inputs

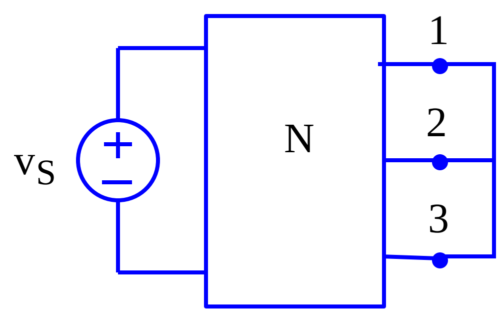
Superposition can be applied, thus, the voltages between any pair of terminals on the axis of symmetry is :

$$v_{ij} = v'_{ij} + v''_{ij}$$

However, symmetry conditions yield:

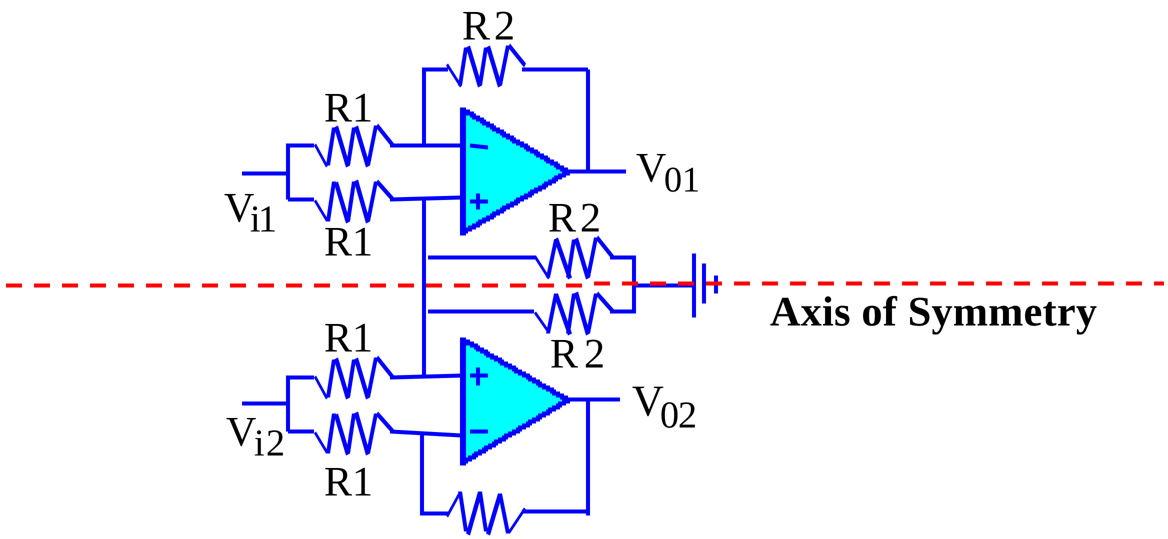
$$v'_{ij} = -v''_{ij}$$

Therefore, the equivalent half circuit becomes

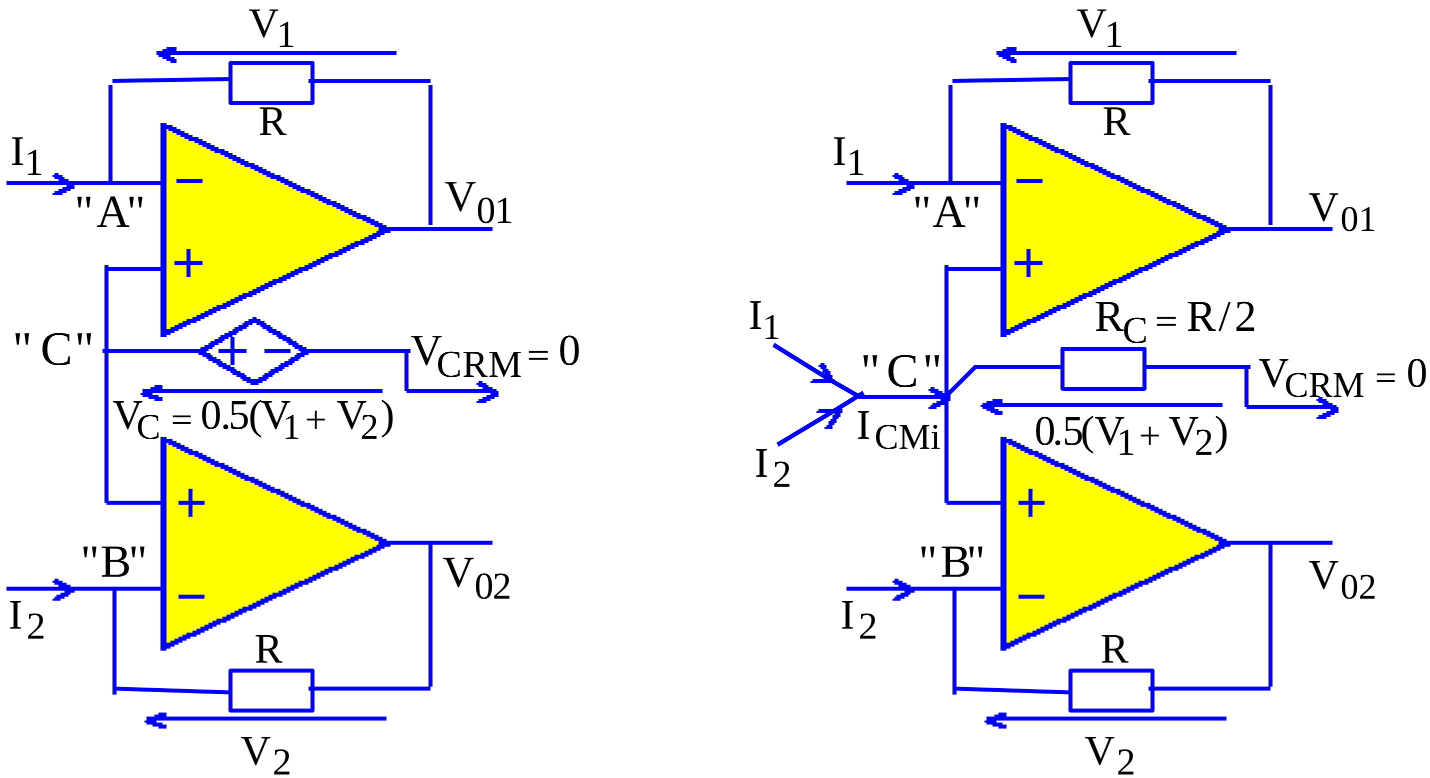


Antisymmetrical Input (Differential-mode)

An example of a Symmetric Circuit



What are the differential gain, and the common mode gain ?



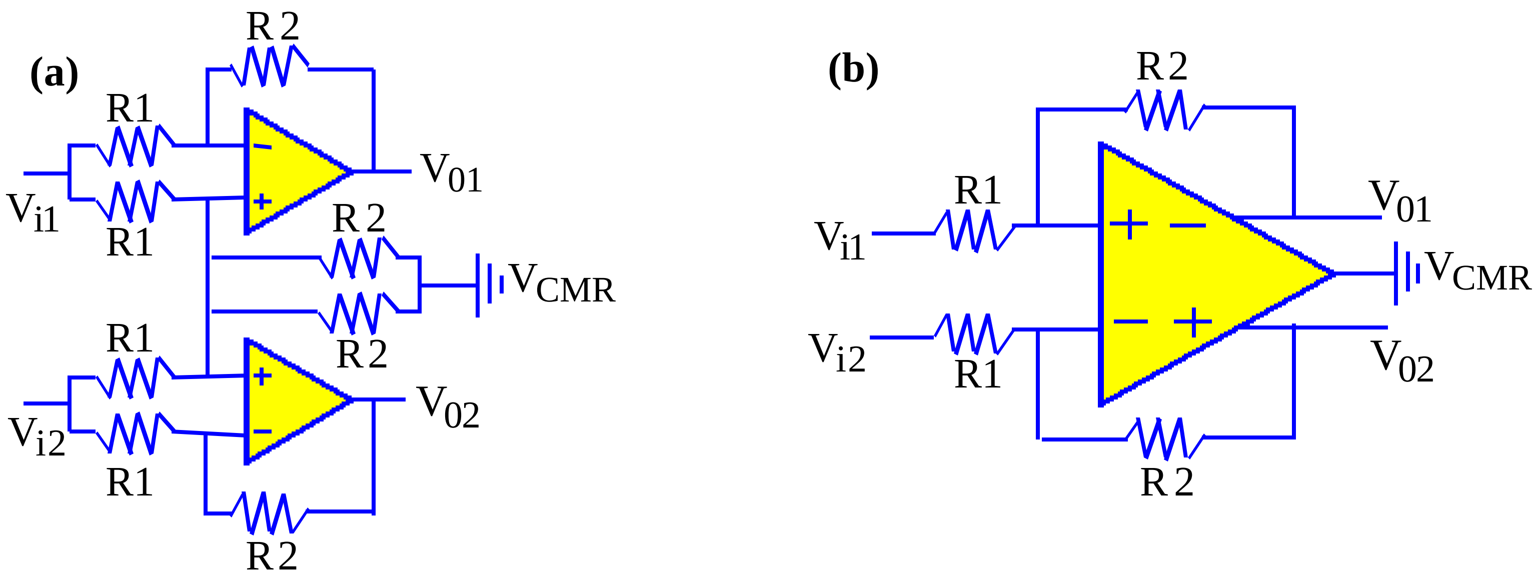
Basic principle of FB architecture with only CM feedforward path.

$$V_C(s) = \frac{V_1(s) + V_2(s)}{2}$$

is inserted between node "C" and a reference node. Note that a $V_C(s)$ voltage is controlled by the common-mode value of the feedback voltages $V_1(s)$ and $V_2(s)$. Assuming that the input of each amplifier are virtually shorted it allows the output voltages $V_{01}(s)$ and $V_{02}(s)$ to be fully balanced around a constant voltage V_{CMR} (= 0) applied to the reference node. They are given as follows:

$$V_{01}(s) = -\frac{V_1(s) - V_2(s)}{2} = -V_{02}(s).$$

Example # 1:



An inverting FB amplifier with (a) CM feedforward, and (b) CM feedback

In principle, both circuits realize the same differential and common - mode transfer functions. However, the new FB inverting amplifier in Fig. (a) can be realized more simply, i.e, with only ordinary opamps, than the conventional approach shown in Fig. (b). Its stability is also improved because a CM feedback is not required.

Examples: #2

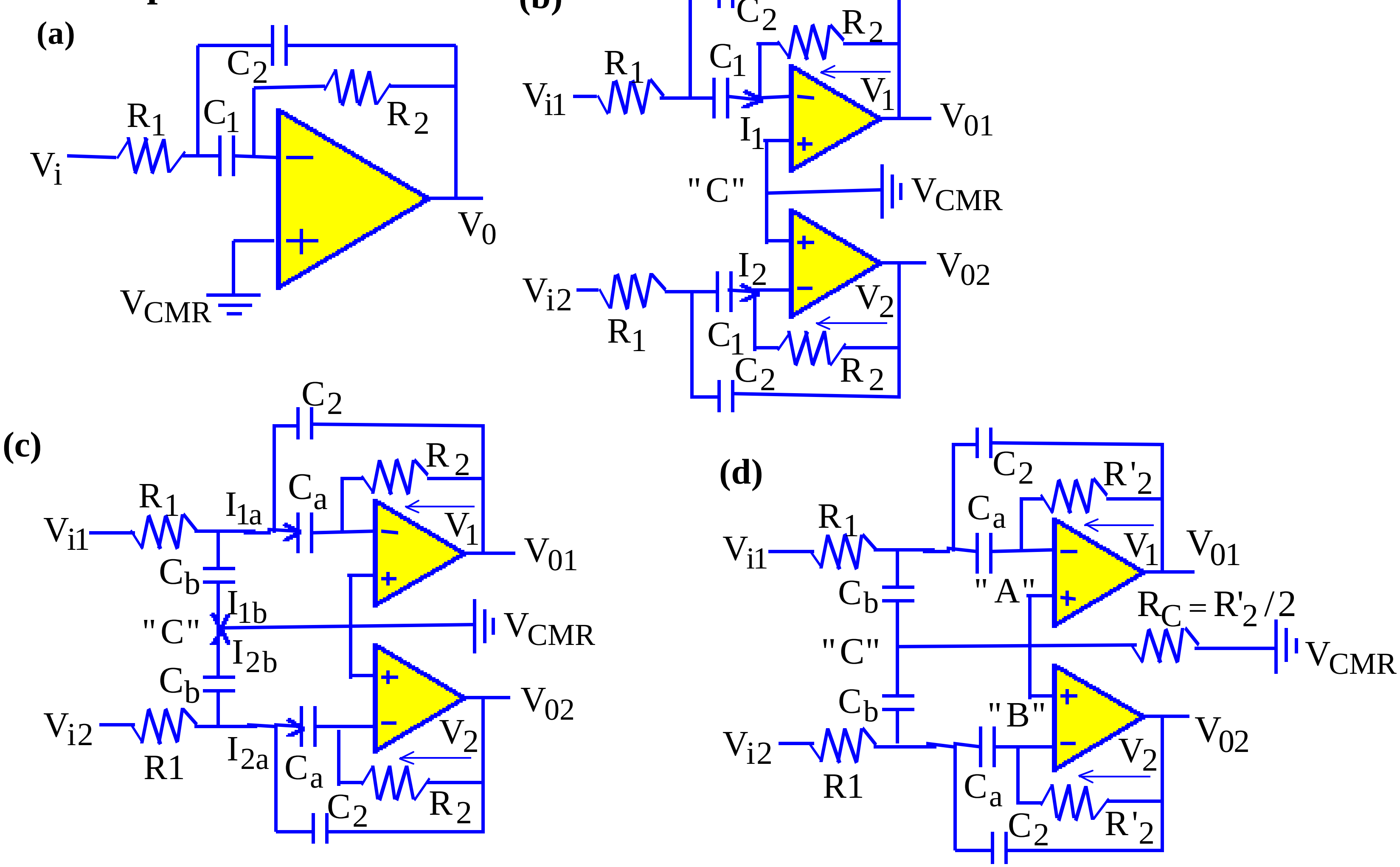


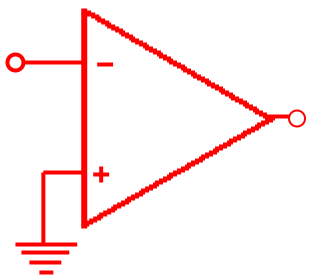
Fig. (a) A second-order single ended prototype filter. (b) a differential filter architecture, (c) a modified differential filter architecture, (d) a fully balanced filter architecture.

The effective design plan under the proposed architecture is summarized in the following example :

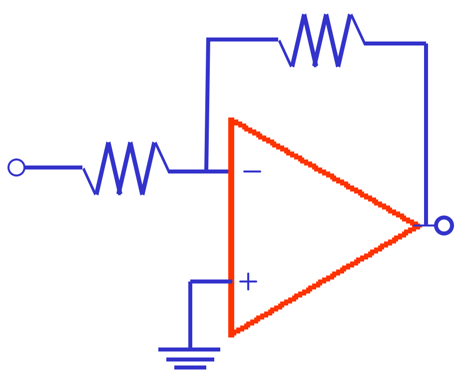
(A) A single - ended version of a bandpass filter shown in Fig.(a) is designed.

(B) A differential but not balanced version of the system shown in Fig.(b) is created. It is achieved by simple duplication of the circuit shown in Fig.(a).

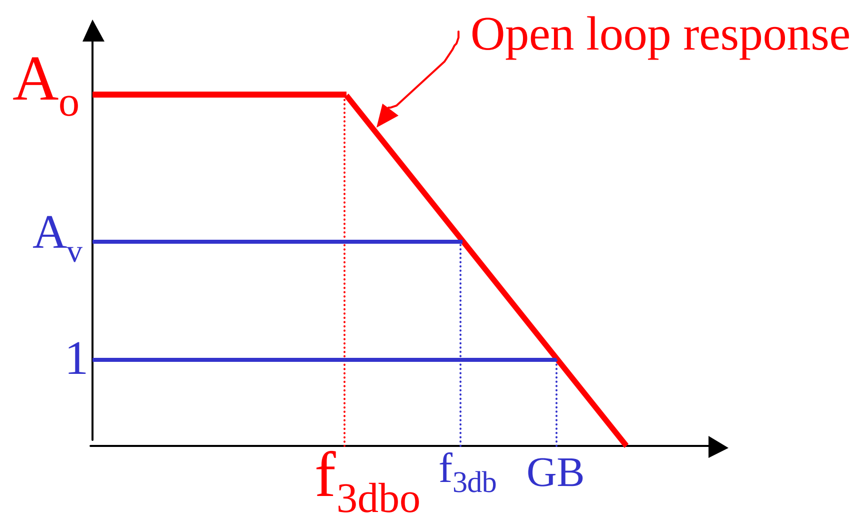
Gain-Bandwidth Product
and
Closed Loop Gain



Open-Loop



Closed-Loop



Do Real Op Amp Have Infinite Bandwidth?

$$A(s) \cong \frac{A_o}{1 + s/\omega_p} = \frac{A_o \omega_p}{s + \omega_p} = \frac{GB}{s + \omega_p}$$

Why do Op Amp have a dominant pole?

-- Stability Consideration

-- i.e. LM741 Op Amp

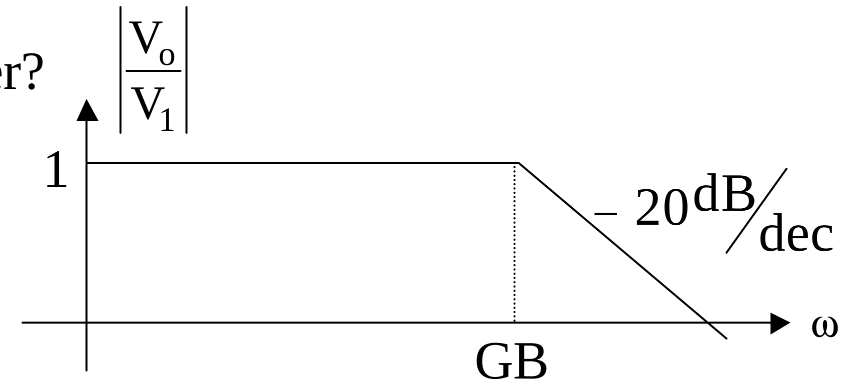
$$A_o = 2 \times 10^5 \, V/V, \; \omega_p = 2\pi \times 5 \, r/s$$

$$GB = 2\pi \times 10^6 \, r/s$$

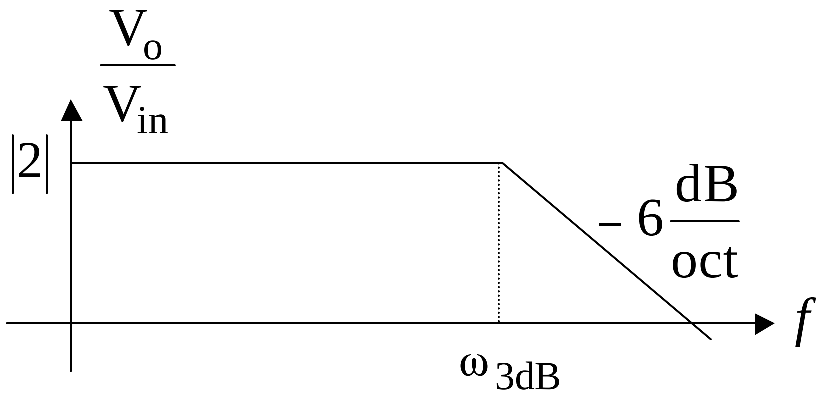
How does this frequency dependence affect the Op Amp circuit performance?

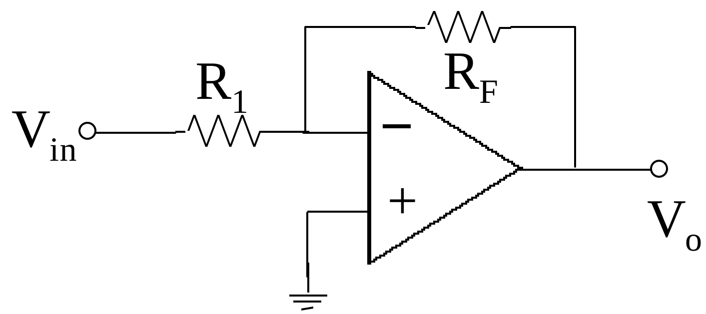
Let's consider the voltage follower?

$$\frac{V_o}{V_{in}} = \frac{A}{1 + A}\Bigg|_{A \cong \frac{GB}{s}} = \frac{GB}{s + GB}$$



Problem: Design an Inverter Amplifier of Voltage Gain of -2 to Operate from DC to 1 MHz.





$$\omega_{3dB} = \frac{GB}{1 + \frac{R_F}{R_1}}$$

$$GB\Big|_{\frac{R_F}{R_1} = 2} = 3\omega_{3dB} = 3\text{MHz}$$

The commercial Op Amp must have a GB of at least 3MHz, say LF353.

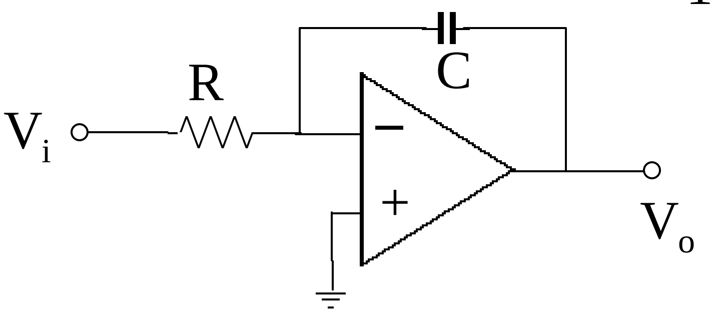
Reader:

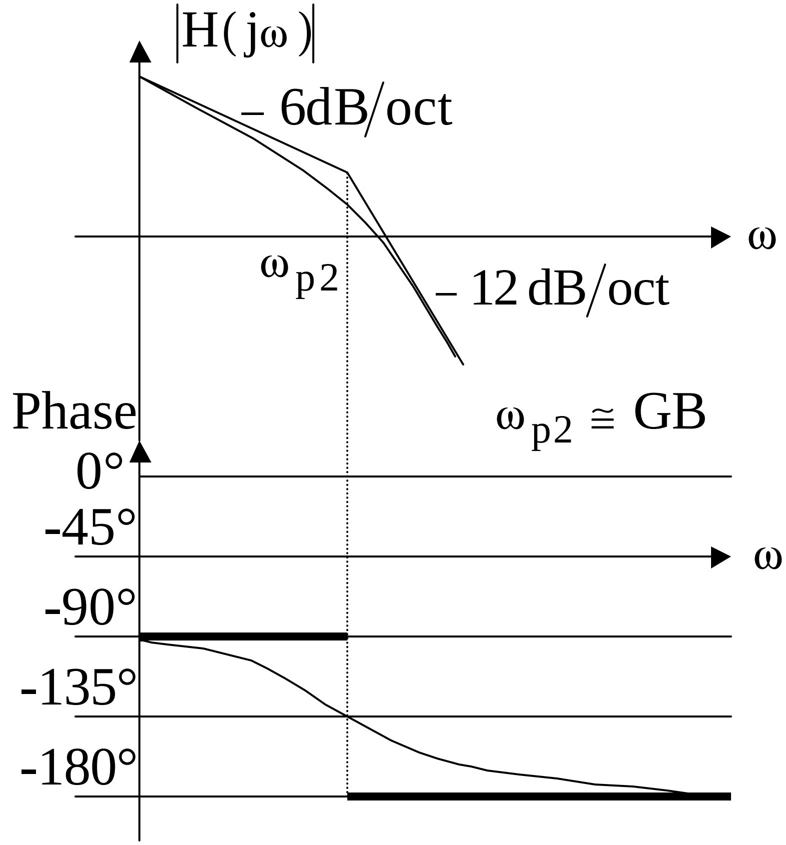
If an Op Amp has GB=1MHz, what is the ω_{3dB} for a gain of $|3|$, for both inverting and non-inverting amplifiers?

GB Effects on Basic Building Blocks

- Inverter Integrator

$$A(s) \cong \frac{GB}{s} \qquad H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{-\dfrac{Z_F}{Z_1}}{1 + \dfrac{s}{GB\left(1 + \dfrac{Z_R}{Z_1}\right)}}$$





$$\omega_{p2} \cong GB$$

$$H(s) = \frac{-\dfrac{1}{sRC}}{1 + \dfrac{s}{GB}\left(1 + \dfrac{1}{sRC}\right)} = \frac{-1}{sRC + \dfrac{s}{GB}(sRC + 1)}$$

$$H(s) = \frac{-GB/1 + RCGB}{s\left(1 + \dfrac{s}{GB + 1/RC}\right)} = \frac{-K}{s\left(1 + s/\omega_{p2}\right)}$$

$$H(j\omega) = \frac{-K}{j\omega - \dfrac{\omega^2}{\omega_p}} \quad ; \quad Q_I \equiv \frac{X(j\omega)}{R_e(j\omega)}$$

Figure of Merit

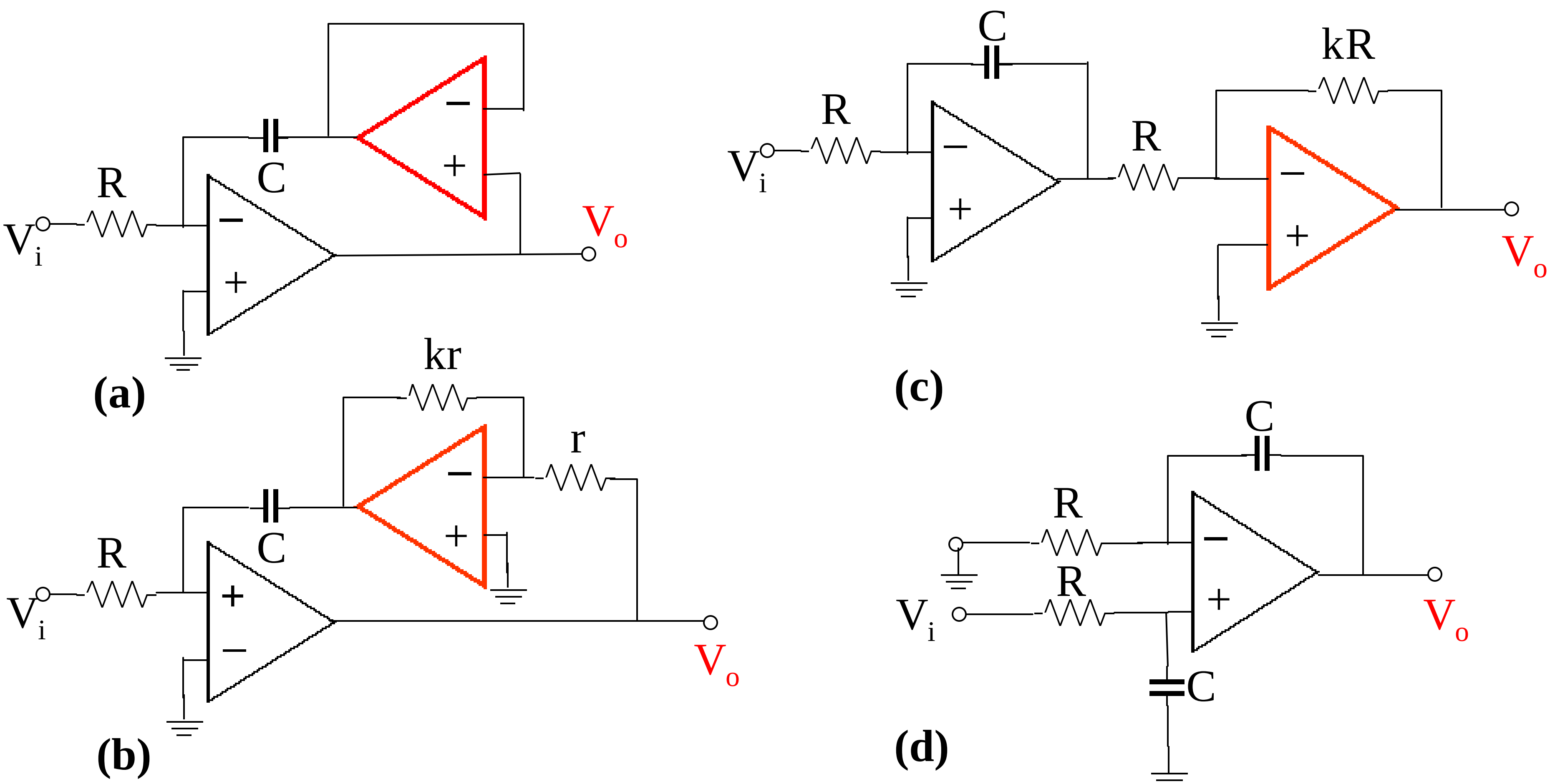
$$Q_I = \frac{\omega\omega_p}{\omega^2} = -\frac{\omega_p}{\omega} = -|A(j\omega)|$$

Ideally

$$Q_I \rightarrow \infty$$

Exercises

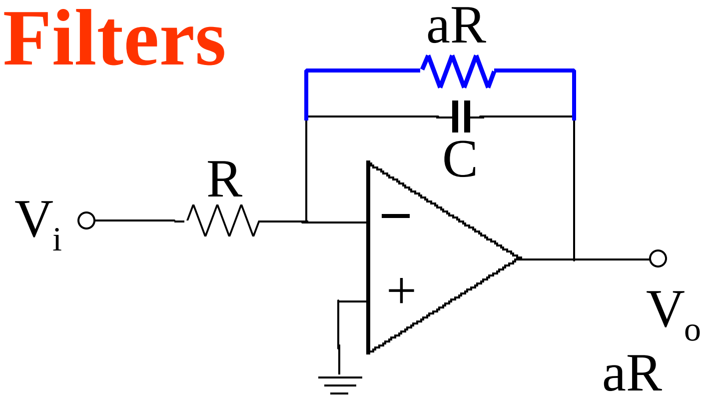
Determine the transfer function of the following integrators assuming $A(s) \cong \frac{GB}{s}$, identical op amps. Also obtain the quality factors (Q_I's for each one.

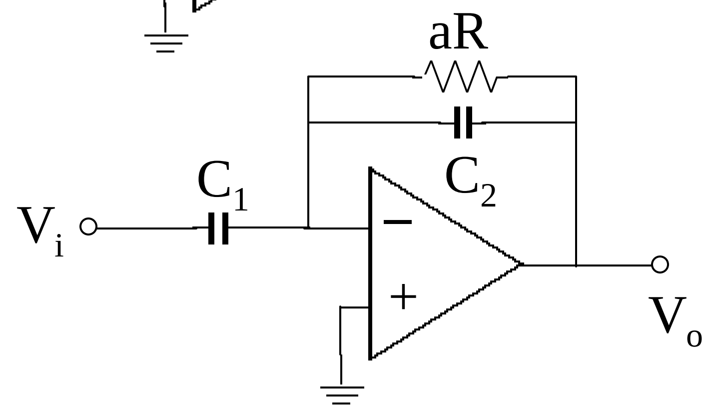


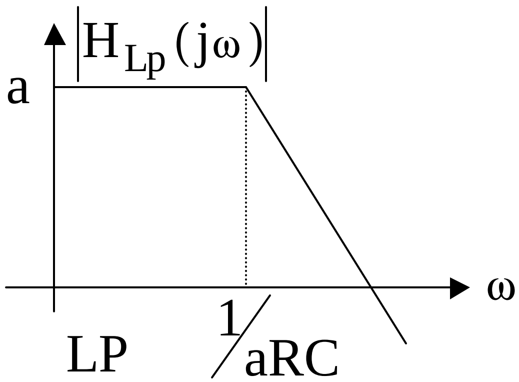
First-Order Filters

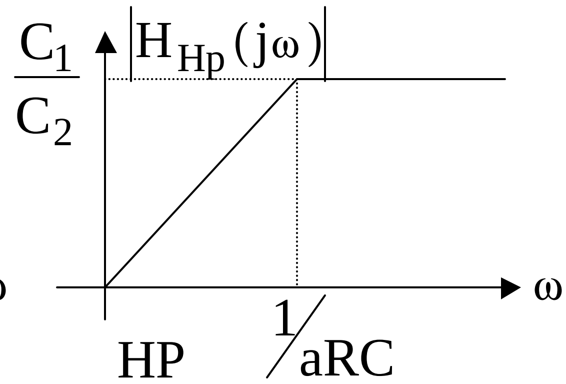
Low-Pass $$H_{Lp}(s) = \frac{-a}{1 + saRC}$$

High-Pass $$H_{Hp}(s) = \frac{-saRC_1}{1 + saRC_2}$$



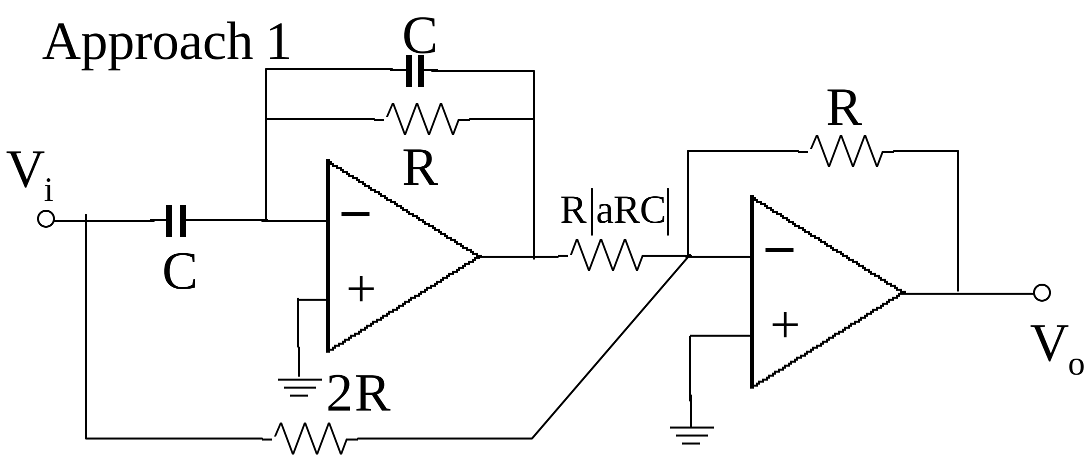




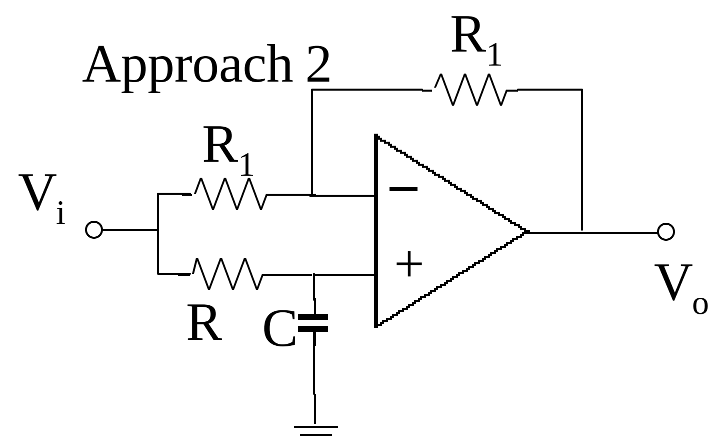


All-Pass $$H_{AP}(s) = +\frac{1}{2}\frac{s-b}{s+b} = -\frac{1}{2} + \frac{s}{s+b}$$

Approach 1

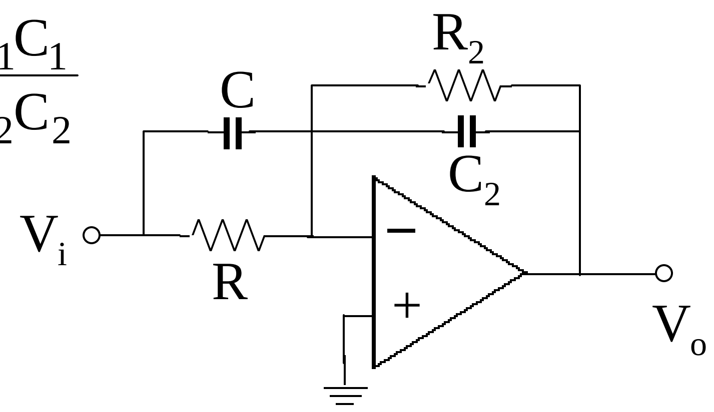


Approach 2



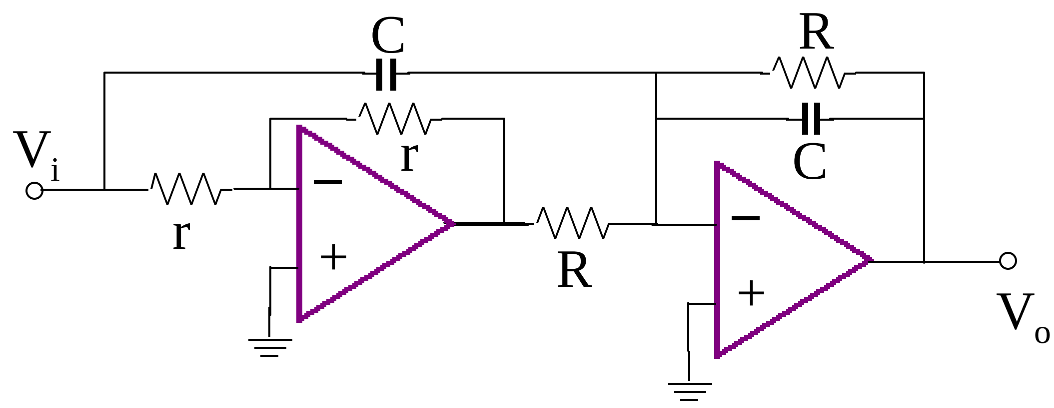
One-Pole/One-LHP-Zero

$$H(s) = -\frac{R_2}{R_1}\frac{1 + sR_1C_1}{1 + sR_2C_2}$$



One-Pole/One-RHP-Zero

$$H(s) = \frac{1 - RCs}{1 + RCs}$$

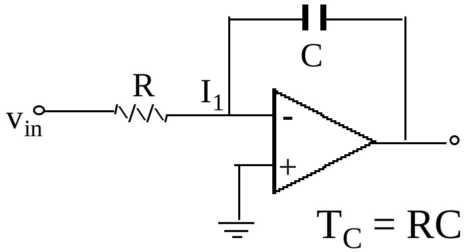
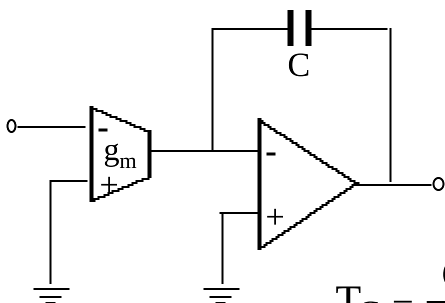
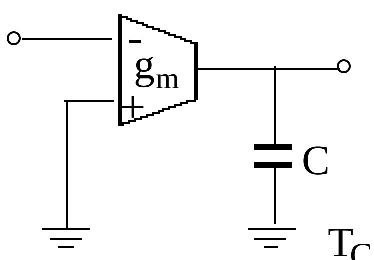


Exercise--

Design a first-order filter with one pole at 1KHz and a RHP zero at 2KHz.

Plot magnitude and phase vs. ω.

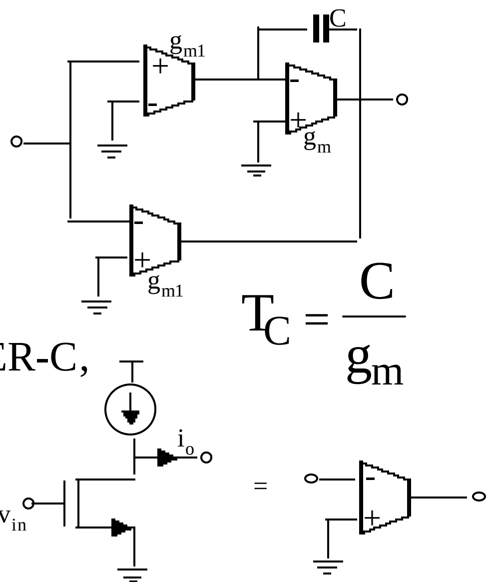
COMPARATIVE VIEW OF INTEGRATORS

Implementation	Remarks
MOSFET-C,  $T_C = RC$ $T_C = \frac{C}{K(V_{C1} - V_{C2})}$	• Non-ideal op amp effects i.e., A_o, GB • Frequency limitations $\omega_o = \frac{1}{T_C} < GB, \quad \text{i.e.,} \ \omega_o \le \frac{GB}{30}$ • Need of buffered op amp • Limited MOSFET's gate voltage
MIXED  $T_C = \frac{C}{g_m}$	• Increased tunability • No loading effect • Still GB effects • No parasitic capacitance effects
OTA-C  $T_C = \frac{C}{g_m}$	• Need linear components • Excellent frequency response • MOSFET OTA tuning range of several octaves • BiCMOS OTA wider tuning range • Suffer parasitic capacitance effects

INTEGRATORS

Implementation	Remarks

Implementation

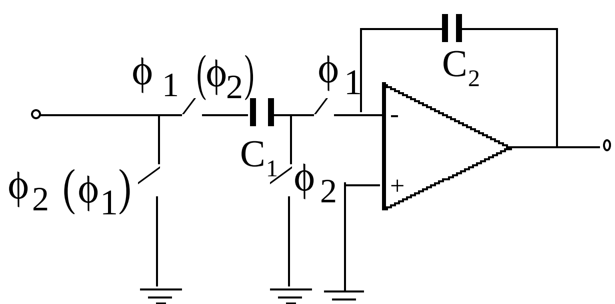


$$T_C = \frac{C}{g_m}$$

INVERTER-C,

Remarks

- Allows use of simple transconductors.
- Uses local feedback to reduce effect of parasitics.
- Open possibilities for new filter implementations.



SC

$$T_C = \frac{1}{f_c}\frac{C_2}{C_1}$$

- Time constant is set by a capacitor ratio and the clock frequency.
- GB effects can be severe at high frequencies.
- Switches feed through effects.
- LV not suitable to turn on switches.

Conclusions

- OTA and Op Amp have different properties and different applications
- Amplifiers non-idealities can degrade performance drastically and limit their applications.
- Fully symmetrical circuits are very useful and can be implemented using single-ended amplifiers providing they are properly connected