ECEN 622: Analog Filters

Homework Assignment #2

By Ramy Ahmed

Problem 1:

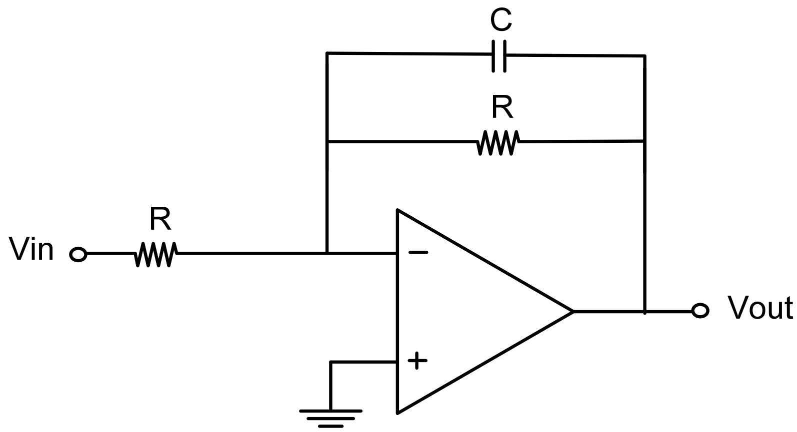


Fig. 1 First-Order Low Pass Filter

a) For an op-amp with finite gain-bandwidth product (GBW), the gain of the low-pass filter (lossy integrator) in Fig. 1 can be derived as follows:

$$H(s) = \frac{\mathrm{Vout(s)}}{\mathrm{VIN(s)}} = \frac{-\mathrm{ZF/R}}{1 + \frac{1}{\mathrm{A(s)}}(1 + \frac{\mathrm{ZF}}{\mathrm{R}})} \qquad \mathrm{V/V}$$

where

$$ZF = \frac{\mathrm{R}}{1 + \mathrm{sCR}} \qquad \Omega$$

Then,

$$H(s) = \frac{\frac{-\mathrm{R}/(1+\mathrm{sCR})}{\mathrm{R}}}{1 + \frac{1}{\mathrm{A(s)}}\left(1 + \frac{\mathrm{R}/(1+\mathrm{sCR})}{\mathrm{R}}\right)} \qquad \mathrm{V/V}$$

$$= \frac{\frac{-1}{(1+\mathrm{sCR})}}{1 + \frac{1}{\mathrm{A(s)}}\left(1 + \frac{1}{(1+\mathrm{sCR})}\right)} \qquad \mathrm{V/V}$$

$$= \frac{-1}{1 + \mathrm{sCR} + \frac{1}{\mathrm{A(s)}}(1 + \mathrm{sCR}) + \frac{1}{\mathrm{A(s)}}} \qquad \mathrm{V/V}$$

But,

$$A(s) = \frac{\mathrm{GBW}}{\mathrm{s}} \qquad \mathrm{V/V}$$

Thus,

$$\mathrm{H(s)} = \frac{-1}{1 + \mathrm{sCR} + \frac{\mathrm{s}}{\mathrm{GBW}}(1 + \mathrm{sCR}) + \frac{\mathrm{s}}{\mathrm{GBW}}} \qquad \mathrm{V/V}$$

$$= \frac{-\mathrm{GBW}}{\mathrm{s^2CR} + \mathrm{s(GBW\,CR} + 1) + \mathrm{GBW} + \mathrm{s}} \qquad \mathrm{V/V}$$

Since $GBW\ X\ CR \gg 1$

$$\mathrm{H(s)} \approx \frac{-\mathrm{GBW}}{\mathrm{s^2CR} + \mathrm{s(GBW\,CR)} + \mathrm{GBW} + \mathrm{s}} \qquad \mathrm{V/V}$$

$$= \frac{-\mathrm{GBW}}{\mathrm{s^2CR} + \mathrm{s(GBW\,CR} + 1) + \mathrm{GBW}} \qquad \mathrm{V/V}$$

$$= \frac{-\mathrm{GBW}}{\mathrm{s^2CR} + \mathrm{s(GBW\,CR} + 1) + \mathrm{GBW}} \qquad \mathrm{V/V}$$

$$\boldsymbol{H(s)} \approx \frac{\mathbf{-1}}{(\mathbf{1} + \mathbf{sRC})(\mathbf{1} + \frac{\mathbf{s}}{\mathbf{GBW}})} \qquad \mathbf{V/V}$$

Thus, the finite GBW of the op-amp introduced an additional pole at the op-amp unity gain frequency.

The dominant pole is given by

$$wp = \frac{1}{\mathrm{RC}} \qquad \mathrm{rad/sec}$$

Therefore, to obtain a cut-off frequency of **1.5915 Hz using C=0.1 nf:**

$$\mathbf{R} = \frac{\mathbf{1}}{\boldsymbol{wp}\mathbf{C}} = \frac{\mathbf{1}}{\boldsymbol{2\pi}(\mathbf{1.5915})\boldsymbol{rad/sec} \times \mathbf{0.1}\,\boldsymbol{nf}} = \mathbf{1\ G\Omega}.$$

Simulation results for AC analysis are given in Fig. 3 to demonstrate the performance of the circuit. The op-amp has been modeled using the macro-model given in Fig. 2 such that the DC gain A_{v0} is 2 X 10^5 V/V && $w3dB = 2\pi \times 4.5\ rad/sec$, so that the GBW is 0.9 MHz.

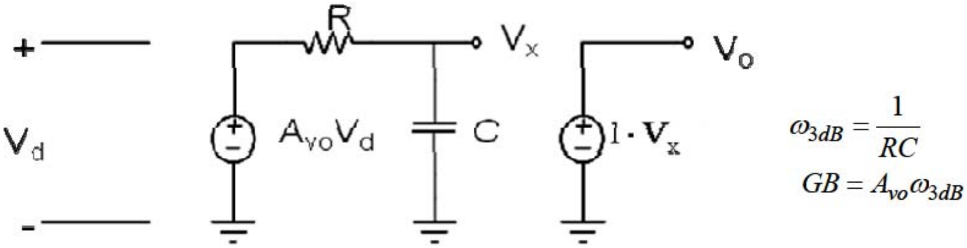
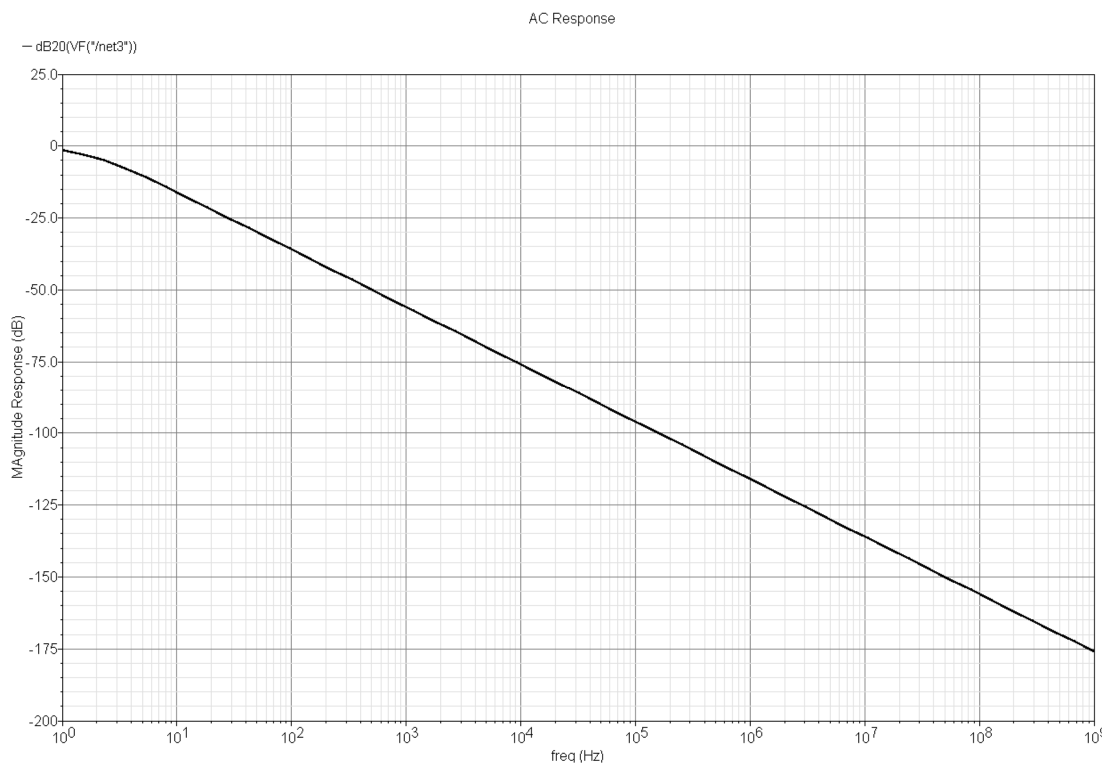
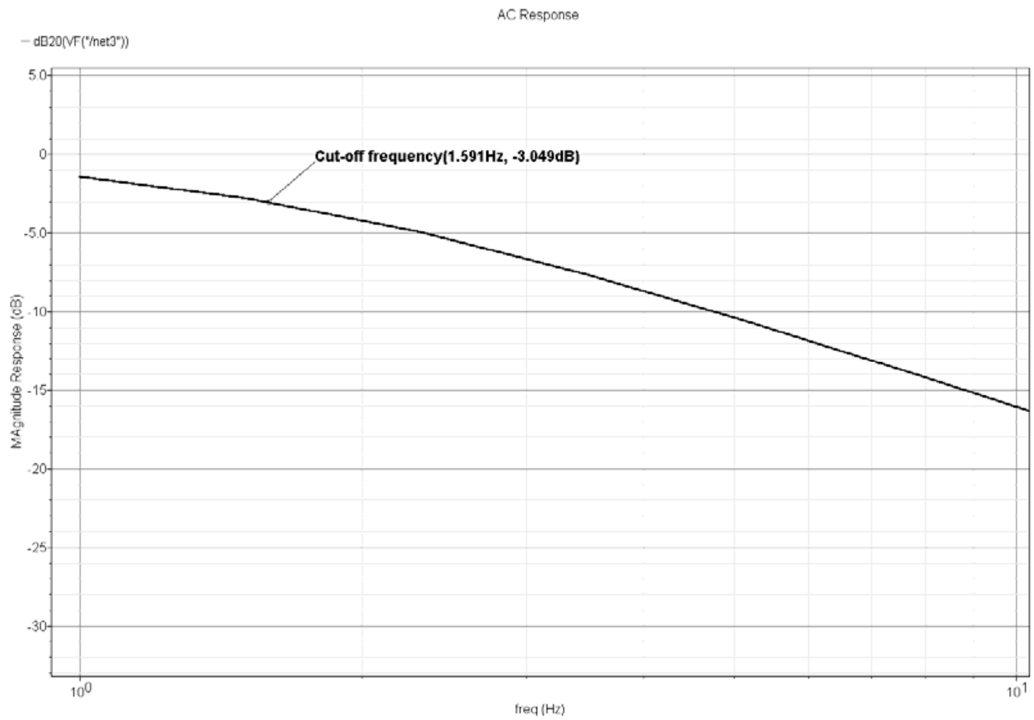


Fig. 2 Op-amp macro-model used in simulations



(a)



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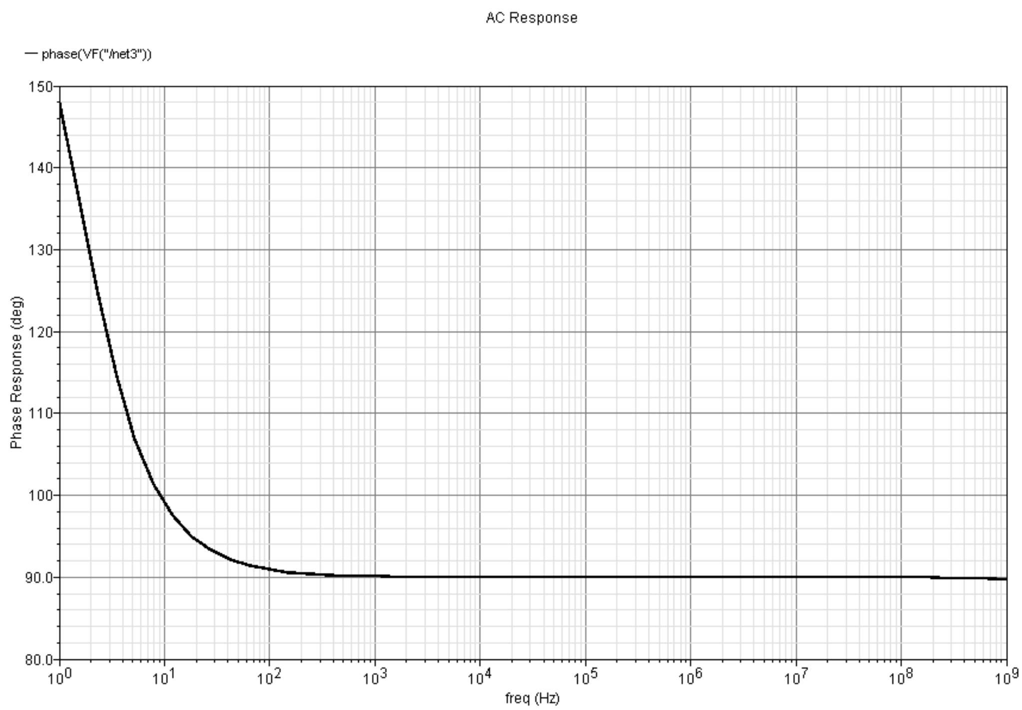


(c)

Fig. 3 Simulated AC Response for the first-order filter in Fig. 1. a) Magnitude Response. b) Magnitude Response magnified over the band of interest. c) Phase Response.

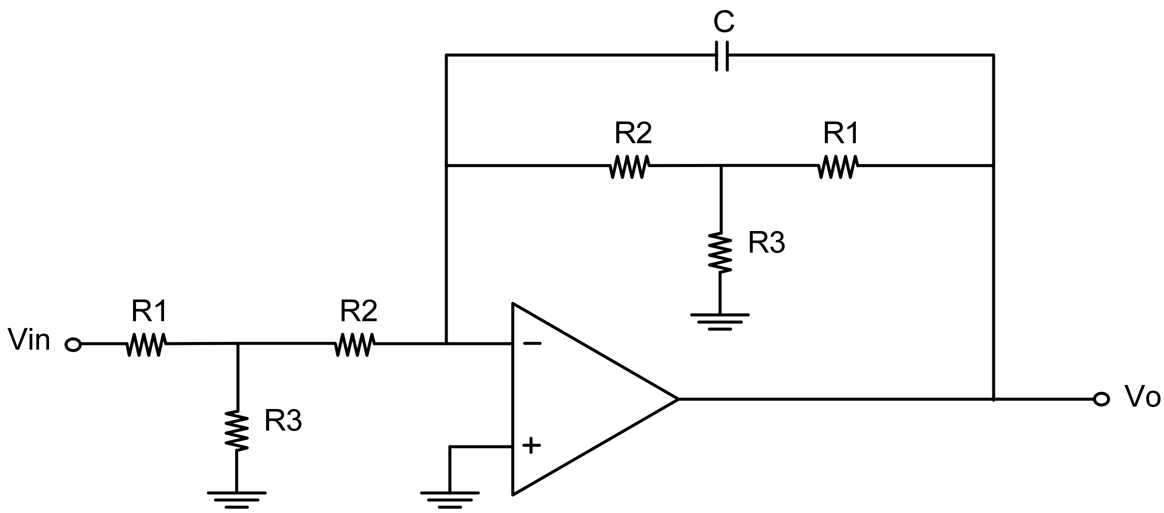


Fig. 4 First-order Low pass filter using resistive T-network

b) The short circuit impedance of the given T-network can be derived as follows:

$$I_2 = \frac{\mathrm{V1}}{\mathrm{Z1} + \frac{\mathrm{Z2\,Z3}}{\mathrm{Z2+Z3}}} \frac{\mathrm{Z3}}{\mathrm{Z2+Z3}} \qquad \mathrm{A}$$

$$= \frac{\mathrm{V1\,Z3}}{\mathrm{Z1\,Z2 + Z2\,Z3 + Z1\,Z3}} \qquad \mathrm{A}$$

Therefore,

$$Zs.c. = \mathrm{Z1 + Z2} + \frac{\mathrm{Z1Z2}}{\mathrm{Z3}} \qquad \Omega$$

On using this T-network instead of the resistors R in the low pass filter of problem 1 a), Z1, Z2, and Z3 are resistances, Z1=R1, Z2=R2, and Z3=R3.

$$Zs.c = \mathrm{Rs.c} = \mathrm{R1 + R2} + \frac{\mathrm{R1R2}}{\mathrm{R3}} \qquad \Omega$$

The equivalent circuit is drawn in Fig. 4. For Rs.c, if $R1 \rightarrow 0$ or $R3 \rightarrow \infty$, then Rs.c will be equal to R2 and the single resistor model is obtained. It is worth noting that this realization is exactly equivalent to the original R resistors-based realization in the ideal sense. This is simply because if the gain/GBW of the op-amp is infinite the negative input terminal will remain at virtual ground all the time, making the current flowing through the T-network at the input independent of the output voltage and hence we can simply obtain the expression for H(s) by replacing R in 1 a) by Rs.c. However, due to finite GBW of actual op-amps, the current flowing through each of the T-networks used in the filter will have dependence on the output voltage due to finite voltage appearing at op-amp negative terminal $-\mathrm{Vo}/\mathrm{A(s)}$. This effect will be derived through the incoming analysis.

The filter transfer function can be derived as follows:

Applying KCL at the negative input terminal of the op-amp,

$$\frac{\mathrm{VIN}}{\mathrm{R1}+\mathrm{R2}+\frac{\mathrm{R1\,R2}}{\mathrm{R3}}}+\frac{\frac{\mathrm{Vout}}{\mathrm{A(s)}}}{\mathrm{R2}+\frac{\mathrm{R1\,R3}}{\mathrm{R1}+\mathrm{R3}}}+\frac{\mathrm{Vout}}{\mathrm{R1}+\mathrm{R2}+\frac{\mathrm{R1\,R2}}{\mathrm{R3}}}+\frac{\frac{\mathrm{Vout}}{\mathrm{A(s)}}}{\mathrm{R2}+\frac{\mathrm{R1\,R3}}{\mathrm{R1}+\mathrm{R3}}}+\mathrm{Vout}\left(1+\frac{1}{\mathrm{A(s)}}\right)\mathrm{sC}=0$$

$$\frac{\mathrm{VIN\,R3}}{\mathrm{R1R3}+\mathrm{R2R3}+\mathrm{R1R2}}+\frac{\frac{\mathrm{Vout}}{\mathrm{A(s)}}(\mathrm{R1}+\mathrm{R3})}{\mathrm{R1R3}+\mathrm{R2R3}+\mathrm{R1R2}}$$

$$=\frac{-\mathrm{Vout\,R3}}{\mathrm{R1R3}+\mathrm{R2R3}+\mathrm{R1R2}}-\frac{\frac{\mathrm{Vout}}{\mathrm{A(s)}}(\mathrm{R1}+\mathrm{R3})}{\mathrm{R1R3}+\mathrm{R2R3}+\mathrm{R1R2}}-\mathrm{Vout}\left(1+\frac{1}{\mathrm{A(s)}}\right)\mathrm{sC}$$

$$\mathrm{VIN\,R3}+\frac{\mathrm{Vout}}{\mathrm{A(s)}}(\mathrm{R1}+\mathrm{R3})$$

$$=-\mathrm{Vout\,R3}-\frac{\mathrm{Vout}}{\mathrm{A(s)}}(\mathrm{R1}+\mathrm{R3})-\mathrm{Vout}\left(1+\frac{1}{\mathrm{A(s)}}\right)\mathrm{sC}(\mathrm{R1R3}+\mathrm{R2R3}+\mathrm{R1R2})$$

$$\mathrm{VIN}\;=-\mathrm{Vout}\;-2\frac{\mathrm{Vout}}{\mathrm{A(s)}}\left(1+\frac{\mathrm{R1}}{\mathrm{R3}}\right)-\mathrm{Vout}\left(1+\frac{1}{\mathrm{A(s)}}\right)\mathrm{sC\,Rs.c}$$

$$\mathrm{H(s)}=\frac{\mathrm{Vout}}{\mathrm{VIN}}\;=\frac{-1}{\left(1+\frac{1}{\mathrm{A(s)}}\right)\mathrm{sC\,Rs.c}+1+\frac{2\left(1+\frac{\mathrm{R1}}{\mathrm{R3}}\right)}{\mathrm{A(s)}}}$$

where

$$A(s)=\frac{\mathrm{GBW}}{\mathrm{s}}\qquad\qquad \mathrm{V/V}$$

$$\mathrm{H(s)}=\frac{-1}{\left(1+\frac{\mathrm{s}}{\mathrm{GBW}}\right)\mathrm{sC\,Rs.c}+1+\mathrm{s}\frac{2\left(1+\frac{\mathrm{R1}}{\mathrm{R3}}\right)}{\mathrm{GBW}}}\qquad\qquad \mathrm{V/V}$$

$$= \frac{-\mathrm{GBW}}{(\mathrm{GBW}+s)sC\,\mathrm{Rs.c} + \mathrm{GBW} + s + s\frac{\left(1+2\frac{\mathrm{R1}}{\mathrm{R3}}\right)}{\mathrm{GBW}}} \qquad \mathrm{V/V}$$

and since

$$GBW \gg \left(1+2\frac{\mathrm{R1}}{\mathrm{R3}}\right)\mathrm{wp}$$

where

$$wp = \frac{1}{\mathrm{C\,Rs.c}} \qquad \mathrm{rad/sec}$$

then,

$$\mathrm{H(s)} \approx \frac{-\mathrm{GBW}}{(\mathrm{GBW}+s)sC\,\mathrm{Rs.c} + \mathrm{GBW} + s} \qquad \mathrm{V/V}$$

$$= \frac{-\mathrm{GBW}}{(\mathrm{GBW}+s)(1+sC\,\mathrm{Rs.c})} \qquad \mathrm{V/V}$$

It can be seen here that the s factor at the denominator that will be neglected is

$$s\frac{\left(1+2\frac{\mathrm{R1}}{\mathrm{R3}}\right)}{\mathrm{GBW}}$$

, which has a higher value than its counterpart in case of R resistors (1-a) which is equal to

$$s\frac{1}{\mathrm{GBW}}$$

Thus, slightly higher requirement is added on the GBW of the op-amp. As illustrated in the simulations, this difference wouldn't make any noticeable change in the resulting plots, since these unit variations are neglected when compared to GBW (~ MHz).

The filter gain is given by:

$$\boldsymbol{H(s) \approx \frac{-1}{(1+sC\,\mathrm{Rs.c})(1+\frac{s}{\mathrm{GBW}})}} \qquad \mathbf{V/V}$$

Thus, from this expression it can be deduced that to achieve the same cut-off frequency as in prob. 1 a), Rs.c needs to be equal to R= 1 **GΩ**.

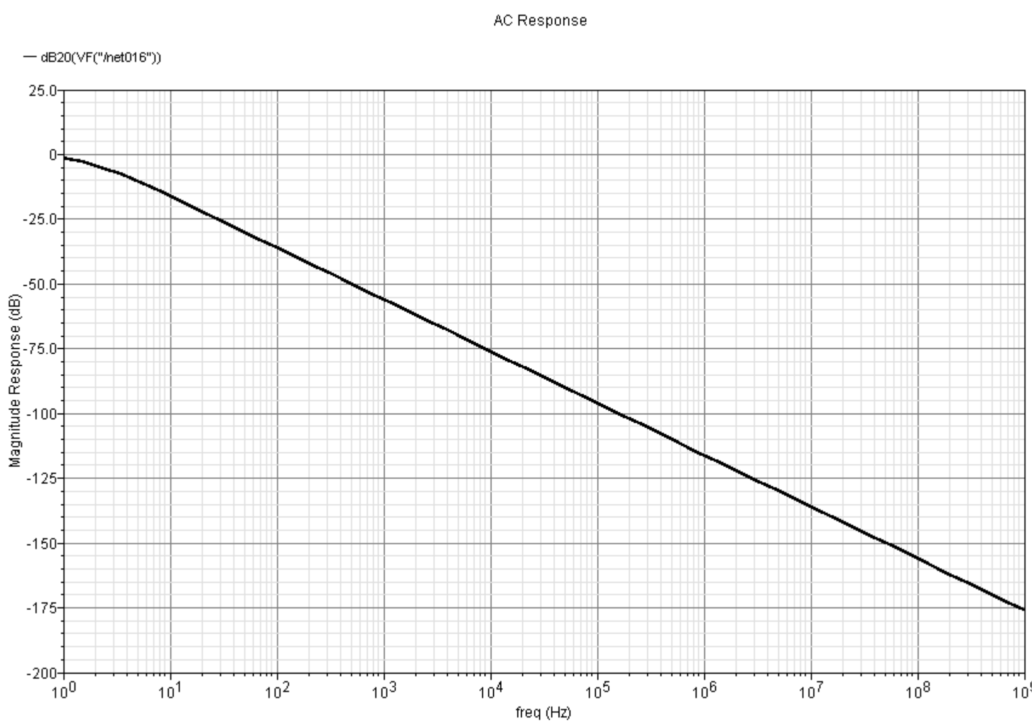
A possible combination can be as follows:

R1 = 400 MΩ
R2 = 200 MΩ
R3 = 200 MΩ.

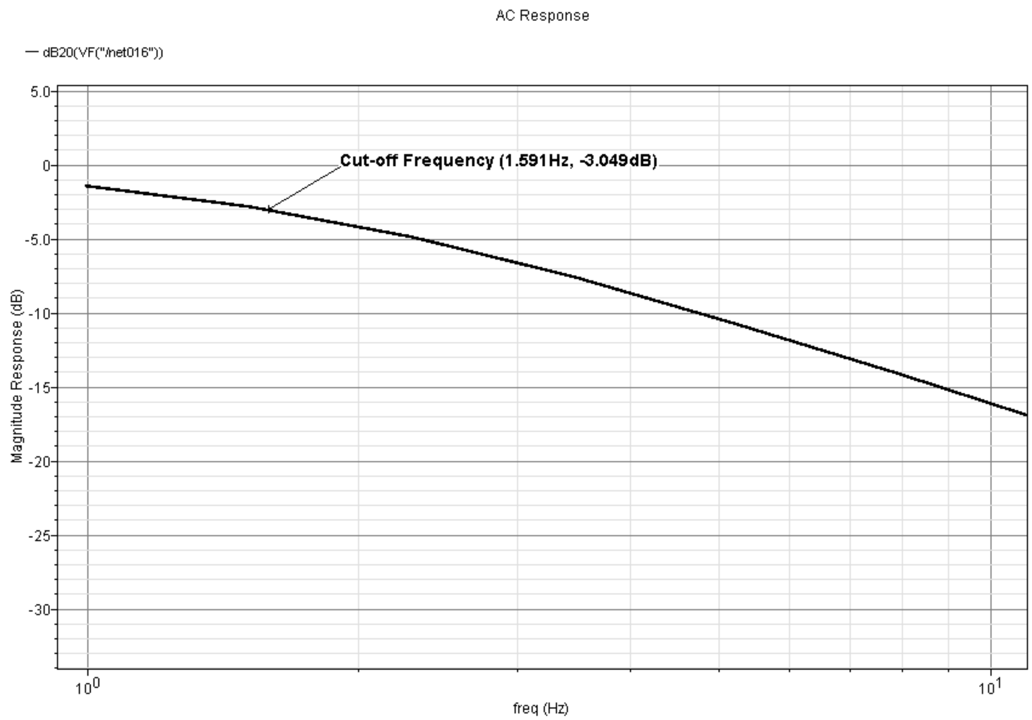
Then,

$$\mathbf{Rs.c} = \mathbf{400\ M\Omega} + \mathbf{200\ M\Omega} + \frac{\mathbf{400\ M\Omega\ \ 200\ M\Omega}}{\mathbf{200\ M\Omega}} = \mathbf{1\ G\Omega}$$

The filter in fig. 4, using the T-network, has been simulated using the same op-amp macro-model in Fig. 2 (used in 1 a) and the resulting bode plots for the AC response are depicted in Fig. 5.



(a)



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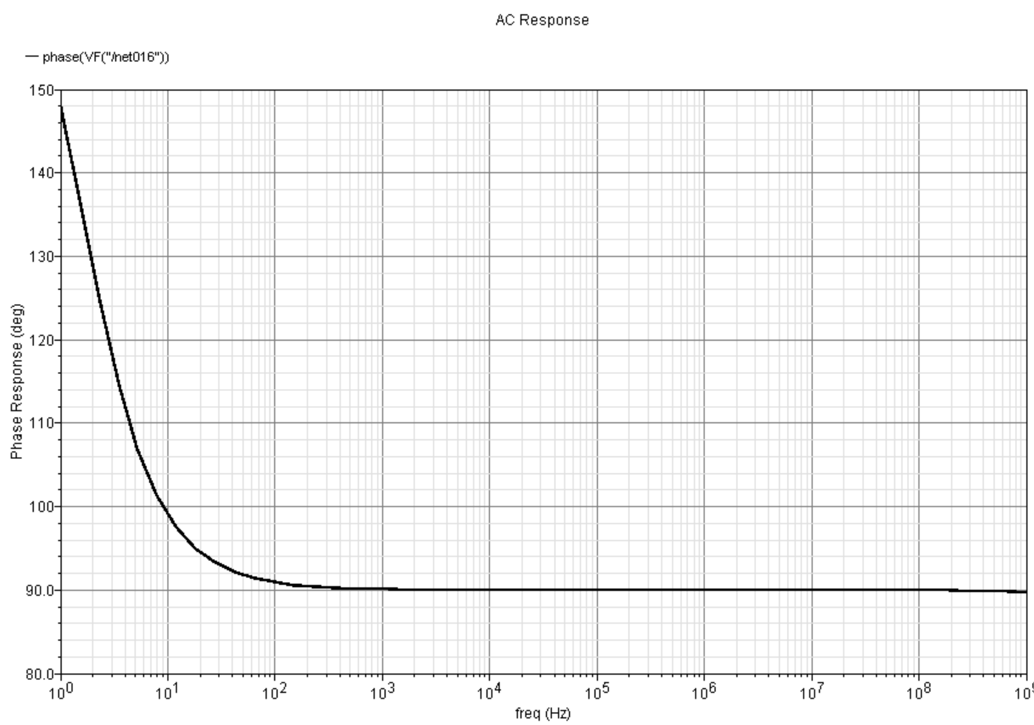
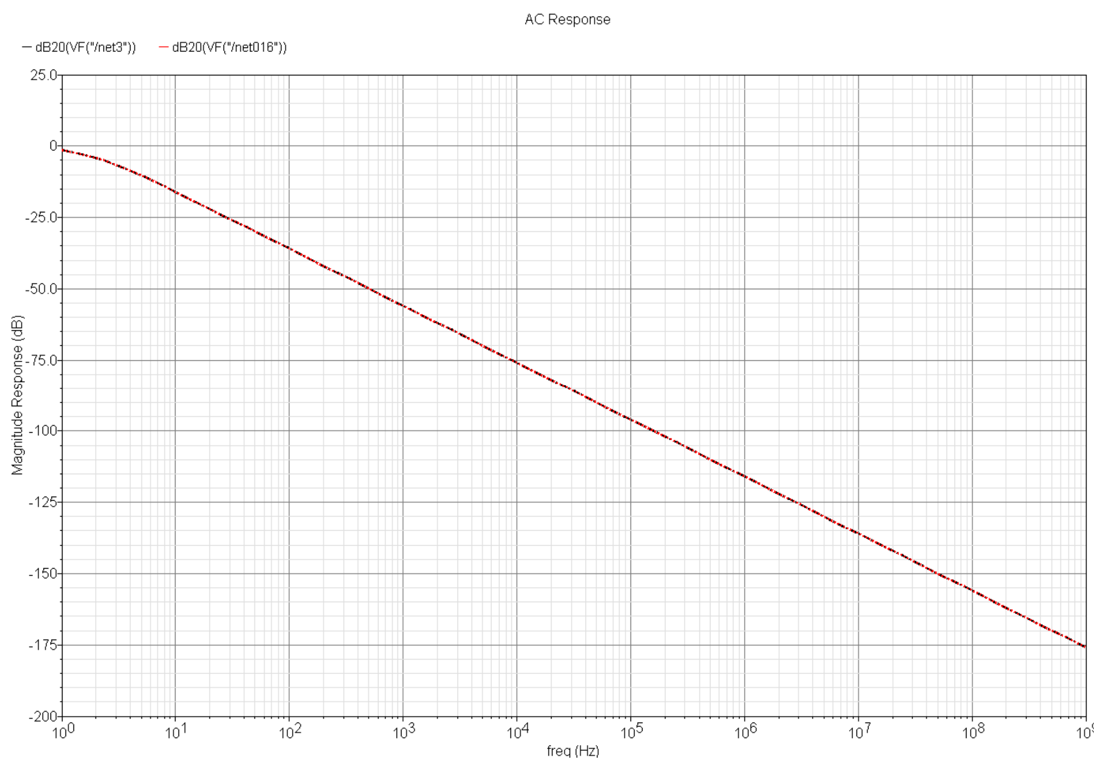


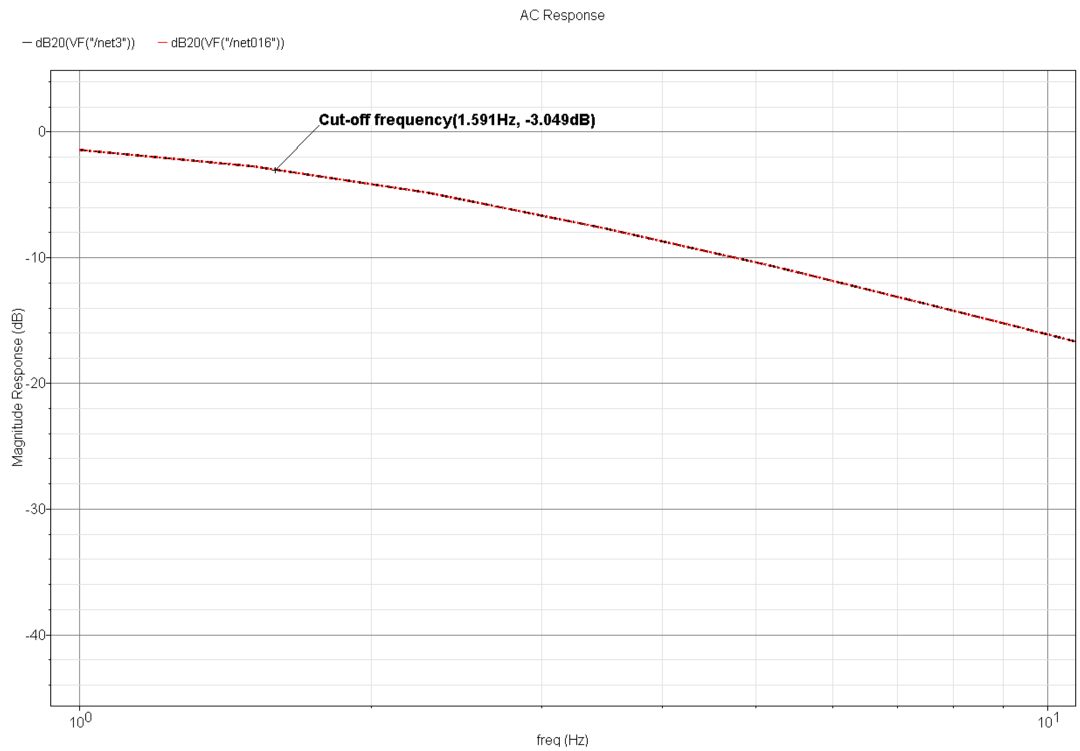
(c)

Fig. 5 Simulated AC Response for the first-order filter in Fig. 4. a) Magnitude Response. b) Magnitude Response magnified over the band of interest. c) Phase Response.

As shown, the response is exactly the same as in case of using R resistors. Fig. 6 shows the plots of the two cases over each other to explore the differences at DC and around the poles. If there is a noticeable error, it should show up near the second pole (around f = GBW), however, the two plots are completely coinciding due to the completely negligible value of the difference compared to the value of GBW.



(a)



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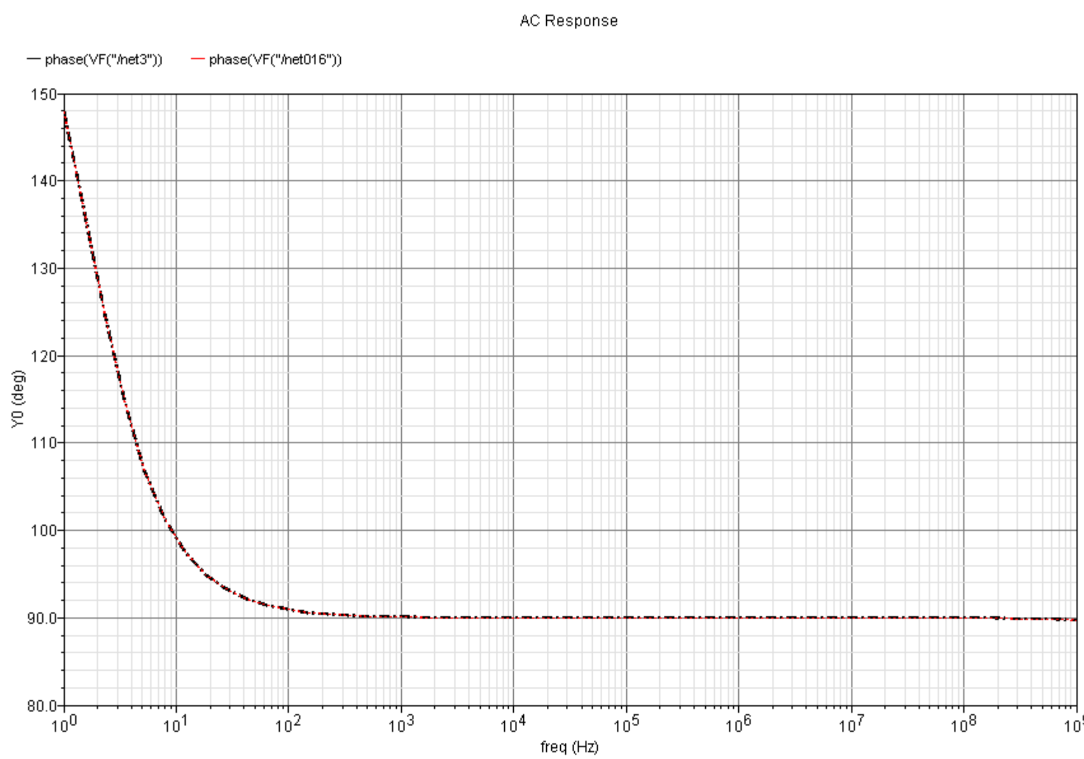


(c)

Fig. 6 Combined Simulated Responses for both filters a) Magnitude Response. b) Magnitude Response magnified over the band of interest. c) Phase Response.

The key advantages offered by using the T-network instead of the resistors R are as follows:

i- Avoiding the need for a prohibitively large resistors (e.g. ~ 1 **GΩ**) to obtain large time constants needed for low frequency filters. The required equivalent resistance can be achieved using a set of resistances with lower values.

ii- R3 is grounded resistor, more amenable for monolithic integration. Thus, we have two floating resistors R1 and R2 and a grounded resistor R3 such that the sum of the three resistors is less that R=1 GΩ and one of the three resistors is grounded instead of a single (bulky) floating resistor of 1 GΩ in the original case.

iii- The sensitivity of the cut-off frequency to the variations of R1, R2, and R3 is less than the sensitivity in case of R. For the filter in 1 a) using resistances R:

$$wp = \frac{1}{\mathrm{C\,R}} \qquad \text{rad/sec}$$

Thus,

$$\boldsymbol{S_R^{wp} = \frac{\partial wp}{\partial R} \times \frac{\mathrm{R}}{wp} = -1}$$

Whereas, after using the T-network,

$$wp = \frac{1}{\mathrm{C\,Rs.c}} = \frac{1}{(\mathrm{R1} + \mathrm{R2} + \frac{\mathrm{R1R2}}{\mathrm{R3}})\mathrm{C}} \qquad \text{rad/sec}$$

$$\boldsymbol{S_{R1}^{wp} = \frac{\partial wp}{\partial R1} \times \frac{\mathrm{R1}}{wp} = \frac{-1}{(1 + \frac{\mathrm{R2}}{\mathrm{R1}(1 + \frac{\mathrm{R2}}{\mathrm{R3}})})}}$$

$$\boldsymbol{S_{R2}^{wp} = \frac{\partial wp}{\partial R2} \times \frac{\mathrm{R2}}{wp} = \frac{-1}{(1 + \frac{\mathrm{R1}}{\mathrm{R2}(1 + \frac{\mathrm{R1}}{\mathrm{R3}})})}}$$

$$\boldsymbol{S_{R3}^{wp} = \frac{\partial wp}{\partial R3} \times \frac{\mathrm{R3}}{wp} = \frac{1}{\left(1 + \frac{\mathrm{R1} + \mathrm{R2}}{\mathrm{R1R2}}\right)\mathrm{R3}}}$$

Moreover, the statistical variance of the cut-off frequency due to variations in resistances values is smaller due to the different polarities of the sensitivities.

On the other hand, the T-network approach has some drawbacks as well:

i- Slightly increasing the sensitivity to finite op-amp gain, recall that as stated earlier, on using the resistor T-network, the condition on the GBW is

$$GBW \gg \left(1 + \frac{2R1}{R3}\right) wp$$

, whereas in case of using single resistor R, the condition is $\boxed{GBW \gg wp.}$ However, this slight different wouldn't make any noticeable changes in the output due to high values of GBW.

ii- The noise contributed by the T-network is higher than that in case of the resistor R. The input referred noise for each case will be deduced in next question.

To determine the output noise, the easiest way is to calculate the input referred noise power and multiply it by the square of the filter gain. Consider the modified diagram given by Fig. 7, for the filter in 1 a) including the mean square voltage noise sources to model the effect of noise squared voltage.

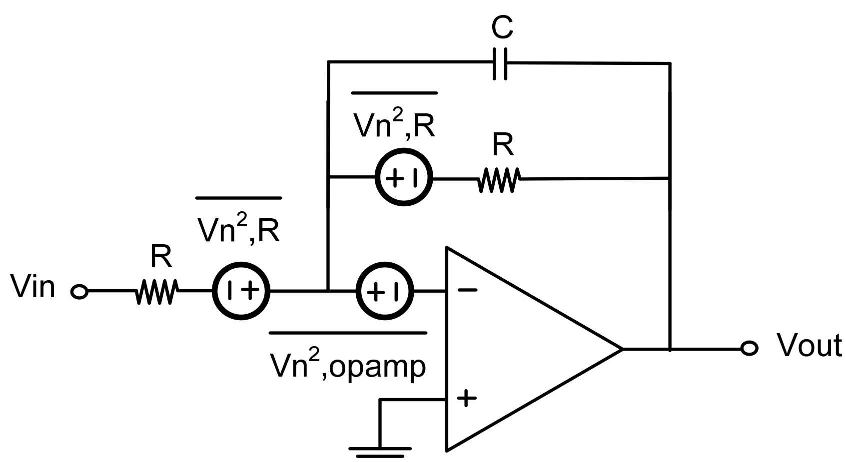


Fig. 7 First-order Active-RC Low pass filter with modeled noise sources.

The input referred noise power is given by:

$$\boldsymbol{\overline{V_{n,in}^2} = \overline{V_{n,R}^2} + \frac{\overline{V_{n,R}^2}}{|H(s)|^2} + \overline{V_{n,opamp}^2}\left(1 + \frac{1}{|H(s)|^2}\right)}$$

where

$V_{n,R}^2 = 4KTR\Delta f$, K is Boltzmann's constant, T is the absolute temperature in Kelvin, R is the value of the resistance R, and Δf is the bandwidth of interest occupied by the noise.

$H(s)$ is the filter transfer function.

$\overline{\boldsymbol{V_{n,opamp}^2}}$, is the input referred noise mean square voltage between the op-amp input terminals.

No flicker noise is considered because no active devices are used in the op-amp macro-modeling. Also, the input referred noise current of the op-amp is neglected here because it is effective only at very high frequencies, whereas this filter is a very low frequency filter.

The total output noise power will be simply:

$$\overline{\boldsymbol{V_{n,out}^2}} = \overline{\boldsymbol{V_{n,in}^2}} \times |\boldsymbol{H(s)}|^2$$

In case of the filter using the T-network in problem 1 b), Fig. 8 is showing the filter with the noise effect modeled as mean square voltage sources.

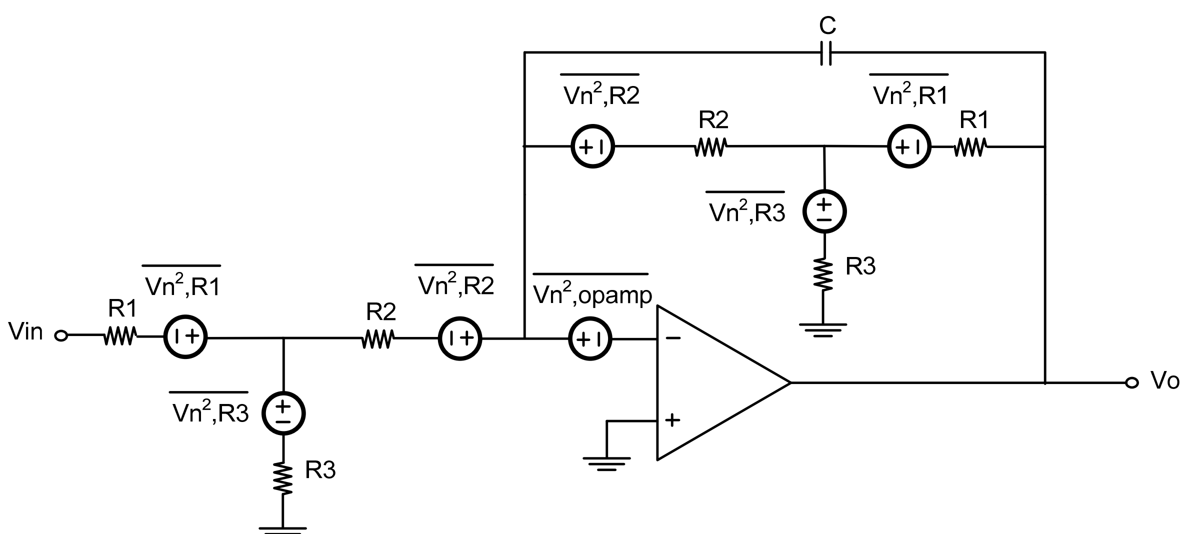


Fig. 8 First-order T-network-based Low pass filter with modeled noise sources.

The input referred noise mean square voltage due to the T-network at the input is given by:

$$\overline{V_{n1,in}^2} = \overline{V_{n,R1}^2} + \overline{V_{n,R3}^2}\frac{Rs.c_1{}^2}{Rs.c_3{}^2} + \overline{V_{n,R2}^2}\left(1 + \frac{Rs.c_1{}^2}{Rs.c_3{}^2}\right)$$

where

$$V_{n,R1}^2 = 4KTR1\Delta f$$

$$V_{n,R2}^2 = 4KTR2\Delta f$$

$$V_{n,R3}^2 = 4KTR3\Delta f$$

$$Rs.c_1 = R1 + R2 + \frac{\mathrm{R1\ R2}}{\mathrm{R3}}$$

$$Rs.c_3 = R3 + R2 + \frac{\mathrm{R3\ R2}}{\mathrm{R1}}$$

The input referred noise mean square voltage due to the T-network in the feedback path is given by:

$$\overline{V_{n2,in}^2} = \frac{\overline{V_{n,R1}^2}}{|H(s)|^2} + \frac{\overline{V_{n,R3}^2}}{|H(s)|^2}|G(s)|^2 + \frac{\overline{V_{n,R2}^2}}{|H(s)|^2}\left(1 + |G(s)|^2\right)$$

where

$$G(s) = \frac{\mathrm{R1\ R2 + R2\ R3 + R1\ R3}}{\mathrm{R3}\left(\mathrm{R2} + \frac{\mathrm{R1}}{\mathrm{A(s)}}\right)}$$

For $A(s) \to \infty$, G(s) reduces to:

$$G(s) = 1 + R1\left(\frac{1}{\mathrm{R2}} + \frac{1}{\mathrm{R3}}\right)$$

The input referred noise mean square voltage resulting from the op-amp noise is given by:

$$\overline{V_{n3,in}^2} = \overline{V_{n,opamp}^2}\left(1 + \frac{Rs.c_1{}^2}{Rs.c_3{}^2} + \frac{1 + |G(s)|^2}{|H(s)|^2}\right)$$

where $\overline{V_{n,opamp}^2}$, is the input referred noise mean square voltage between the op-amp input terminals.

No flicker noise is considered because no active devices are used in the op-amp macro-modeling. Also, the input referred noise current of the op-amp is neglected here because it is effective only at very high frequencies, whereas this filter is a very low frequency filter.

Thus, the total input referred noise mean square voltage is given by:

$$\overline{V_{n,in}^2} = \overline{V_{n1,in}^2} + \overline{V_{n2,in}^2} + \overline{V_{n3,in}^2}$$

$$= \overline{V_{n,R1}^2} + \overline{V_{n,R3}^2}\frac{Rs.c_1^{\,2}}{Rs.c_3^{\,2}} + \overline{V_{n,R2}^2}\left(1 + \frac{Rs.c_1^{\,2}}{Rs.c_3^{\,2}}\right) + \frac{\overline{V_{n,R1}^2}}{|H(s)|^2} + \frac{\overline{V_{n,R3}^2}}{|H(s)|^2}|G(s)|^2 + \frac{\overline{V_{n,R2}^2}}{|H(s)|^2}\left(1 + |G(s)|^2\right) + \overline{V_{n,opamp}^2}\left(1 + \frac{Rs.c_1^{\,2}}{Rs.c_3^{\,2}} + \frac{1 + |G(s)|^2}{|H(s)|^2}\right)$$

$$\overline{V_{n,in}^2} = \overline{V_{n,R1}^2}\left(1 + \frac{1}{|H(s)|^2}\right) + \overline{V_{n,R2}^2}\left(1 + \frac{Rs.c_1^{\,2}}{Rs.c_3^{\,2}} + \frac{1 + |G(s)|^2}{|H(s)|^2}\right) + \overline{V_{n,R3}^2}\left(\frac{Rs.c_1^{\,2}}{Rs.c_3^{\,2}} + \frac{|G(s)|^2}{|H(s)|^2}\right) + \overline{V_{n,opamp}^2}\left(1 + \frac{Rs.c_1^{\,2}}{Rs.c_3^{\,2}} + \frac{1 + |G(s)|^2}{|H(s)|^2}\right)$$

The total output noise power will be simply:

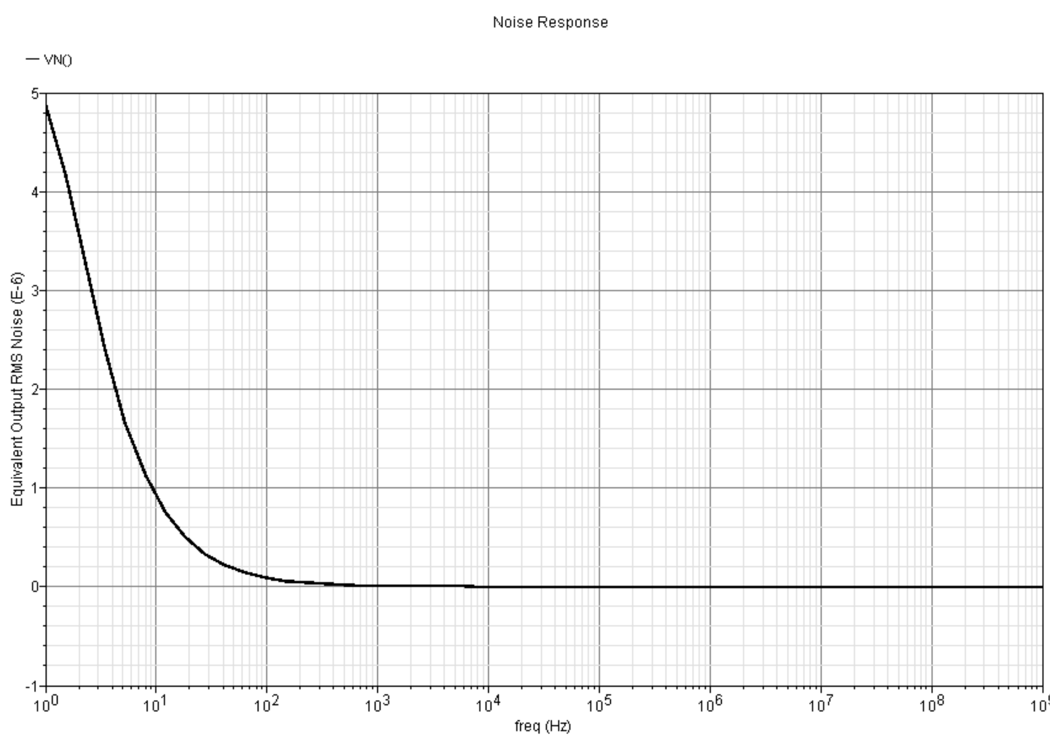
$$\overline{V_{n,out}^2} = \overline{V_{n,in}^2} \times |H(s)|^2$$

Note that the noise mean square voltage components resulting from the resistors in the input T-network and their counterparts in the feedback T-network are added because although these resistors may have same value (e.g. R1 in the input T-network and R1 in the feedback T-network), but their resulting noise is uncorrelated.

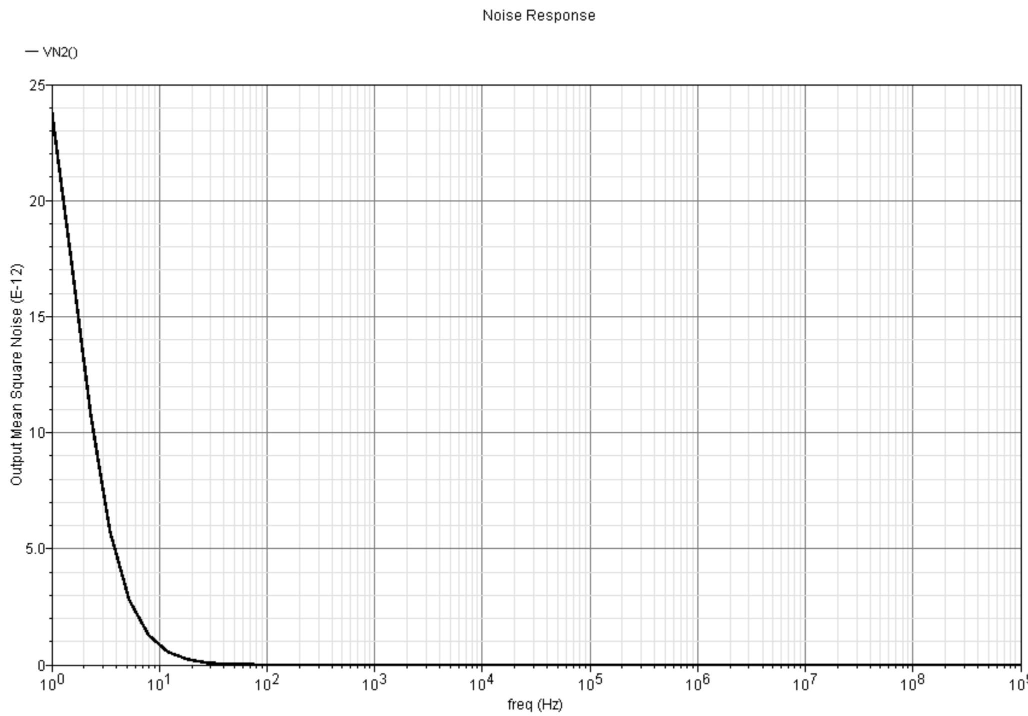
From the expressions of the input referred noise mean square voltages in case of using simple resistors R and in case of using the T-network it is clear that the noise input referred mean square voltages in case of the T-network is higher than the case of the R resistors. Although the total value (sum) of the resistors used in the T-network is lower than the value of R, but

the noise contributed by the resistors R1, R2, and R3 as well as the op-amp experience higher gains yielding higher input referred mean square noise voltage and hence higher output noise because the gain of the two filters is the same.

Figure 9 shows the simulated output noise power and voltage at the output of the filter in Fig. 7. Figure 10 shows the simulated output noise power and voltage at the output of the filter in case of using the T-networks, as in Fig. 8. Simulations demonstrate the above theoretical conclusion. As shown from figures 7 and 8, the output noise in case of the T-network solution (figure 8) is higher than the output noise in case of using resistors R (figure 7) over the band of interest. In both cases, the noise is showing low pass characteristics due to the low pass transfer function of the filter.

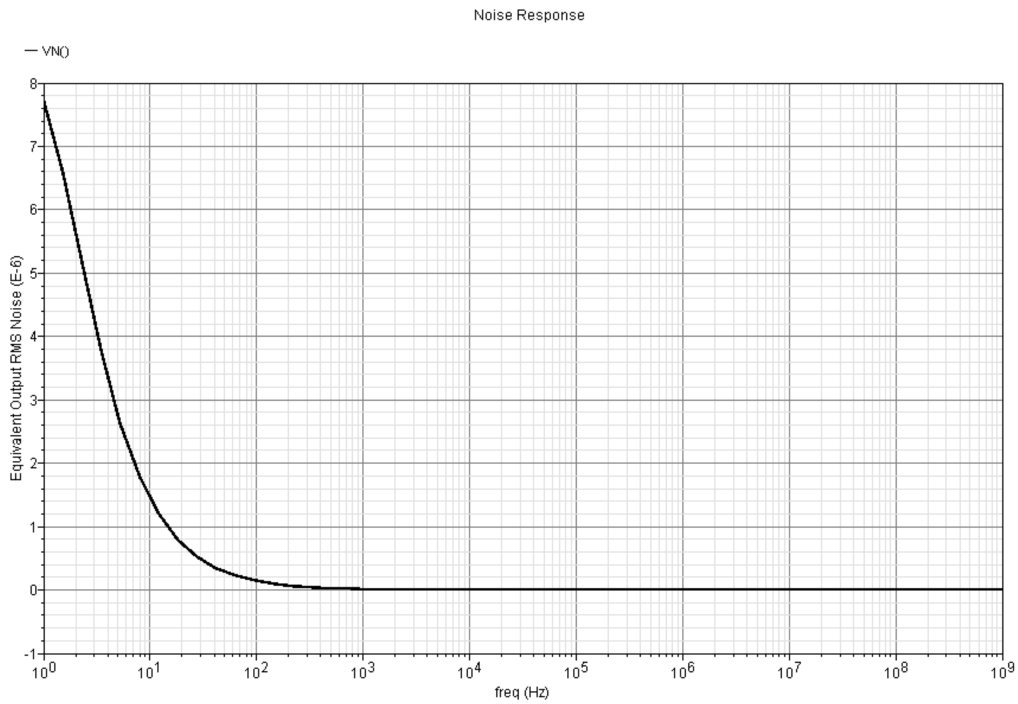


(a)

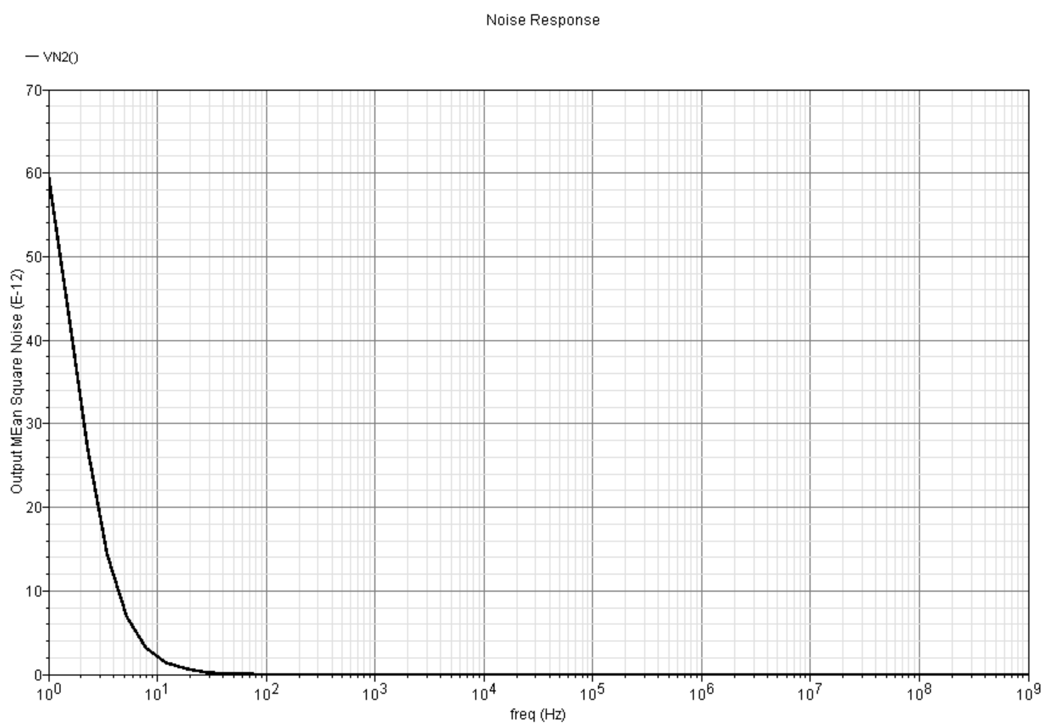


(b)

Fig. 9 Simulated Noise Responses for the LPF in Fig. 7. a) Equivalent Output RMS Noise. b) Equivalent Output Noise Mean Square Voltage.



(a)



(b)

Fig. 10 Simulated Noise Responses for the LPF in Fig. 8. a) Equivalent Output RMS Noise. b) Equivalent Output Noise Mean Square Voltage.

Problem 2:

a) Applying Mason Rule:

$$\mathbf{P1} = \frac{K1}{s}$$

$$\mathbf{P11} = + \frac{-Ko1\ Ko2\ Ko3}{s^3}$$

$$\mathbf{P21} = \frac{-K_Q}{s}$$

Since there are no non-touching loops:

$$\Delta 1 = 1$$

$$\Delta = 1 + \frac{Ko1\ Ko2\ Ko3}{s^3} + \frac{K_Q}{s}$$

$$\mathbf{T(s)} = \frac{C(s)}{R(s)} = \frac{\mathbf{K1/s}}{1 + \frac{Ko1\ Ko2\ Ko3}{s^3} + \frac{K_Q}{s}}$$

$$\mathbf{T(s)} = \frac{C(s)}{R(s)} = \frac{\mathbf{K1}\ s^2}{s^3 + s^2 K_Q + Ko1\ Ko2\ Ko3}$$

b) Applying Mason Rule:

$$\mathbf{P1} = \frac{-K2\ Ko1}{s^2}$$

$$\mathbf{P2} = \frac{-K\ Ko1}{s}$$

$$\mathbf{P11} = \frac{-K_{Q1}}{s}$$

$$\mathbf{P21} = \frac{-Ko1\ Ko2}{s^2}$$

Since there are no non-touching loops:

$$\Delta 1 = \Delta 2 = 1$$

$$\Delta = 1 + \frac{K_{Q1}}{s} + \frac{Ko1\ Ko2}{s^2}$$

$$\mathrm{T(s)} = \frac{C(s)}{R(s)} = \frac{-\frac{K2\ Ko1}{s^2} - \frac{K\ Ko1}{s}}{1 + \frac{K_{Q1}}{s} + \frac{Ko1\ Ko2}{s^2}}$$

$$\mathrm{T(s)} = \frac{C(s)}{R(s)} = \frac{-K2\ Ko1 - K\ Ko1\ s}{s^2 + sK_{Q1} + Ko1\ Ko2}$$

c) <u>Vout1:-</u>

Applying Mason Rule:

$$\mathrm{P1 = K}$$

$$\mathrm{P11} = \frac{-K_Q}{s}$$

$$\mathrm{P21} = \frac{-Ko1\ Ko2}{s^2}$$

Since there are no non-touching loops:

$$\Delta 1 = 1$$

$$\Delta = 1 + \frac{K_Q}{s} + \frac{Ko1\ Ko2}{s^2}$$

$$\mathrm{T(s)} = \frac{C(s)}{R(s)} = \frac{\mathrm{K}}{1 + \frac{K_Q}{s} + \frac{Ko1\ Ko2}{s^2}}$$

$$\mathrm{T(s)} = \frac{C(s)}{R(s)} = \frac{s^2 K}{s^2 + sK_Q + Ko1\ Ko2}$$

➔ **High Pass Filter (HPF)**

Vout2:-

Applying Mason Rule:

$$\mathbf{P1} = \frac{\boldsymbol{K}}{\boldsymbol{s}}$$

$$\mathbf{P11} = \frac{-\boldsymbol{K_Q}}{\boldsymbol{s}}$$

$$\mathbf{P21} = \frac{-\boldsymbol{Ko1\ Ko2}}{\boldsymbol{s^2}}$$

Since there are no non-touching loops:

$$\Delta\mathbf{1} = \mathbf{1}$$

$$\Delta = \mathbf{1} + \frac{\boldsymbol{K_Q}}{\boldsymbol{s}} + \frac{\boldsymbol{Ko1\ Ko2}}{\boldsymbol{s^2}}$$

$$\mathbf{T(s)} = \frac{\boldsymbol{C(s)}}{\boldsymbol{R(s)}} = \frac{\mathbf{K/s}}{\mathbf{1} + \frac{\boldsymbol{K_Q}}{\boldsymbol{s}} + \frac{\boldsymbol{Ko1\ Ko2}}{\boldsymbol{s^2}}}$$

$$\mathbf{T(s)} = \frac{\boldsymbol{C(s)}}{\boldsymbol{R(s)}} = \frac{\boldsymbol{sK}}{\boldsymbol{s^2} + \boldsymbol{sK_Q} + \boldsymbol{Ko1\ Ko2}}$$

➔ **Band Pass Filter (BPF)**

Vout3:-

Applying Mason Rule:

$$\mathbf{P1} = \frac{\boldsymbol{K\ Ko2}}{\boldsymbol{s}}$$

$$\mathbf{P11} = \frac{-\boldsymbol{KQ}}{\boldsymbol{s}}$$

$$\mathbf{P21} = \frac{-Ko1\,Ko2}{s^2}$$

Since there are no non-touching loops:

$$\Delta\mathbf{1} = \mathbf{1}$$

$$\Delta = \mathbf{1} + \frac{KQ}{s} + \frac{Ko1\,Ko2}{s^2}$$

$$\mathbf{T(s)} = \frac{C(s)}{R(s)} = \frac{\frac{K\,Ko2}{s}}{1 + \frac{KQ}{s} + \frac{Ko1\,Ko2}{s^2}}$$

$$\mathbf{T(s)} = \frac{C(s)}{R(s)} = \frac{K\,Ko2}{s^2 + sKQ + Ko1\,Ko2}$$

➔ **Low Pass Filter (LPF)**

Problem 3:

To realize a second order bandpass filter, a two integrator loop filter of class 1a is used. The filter topology is shown in Fig. 11. The circuit diagram is shown in Fig. 12. This filter topology is using a lossy integrator and lossless integrator in a loop with an inverting buffer between them. On taking the output after the first integrator, a bandpass characteristic can be obtained, as deduced in the transfer function for **Vout2** is previous question.

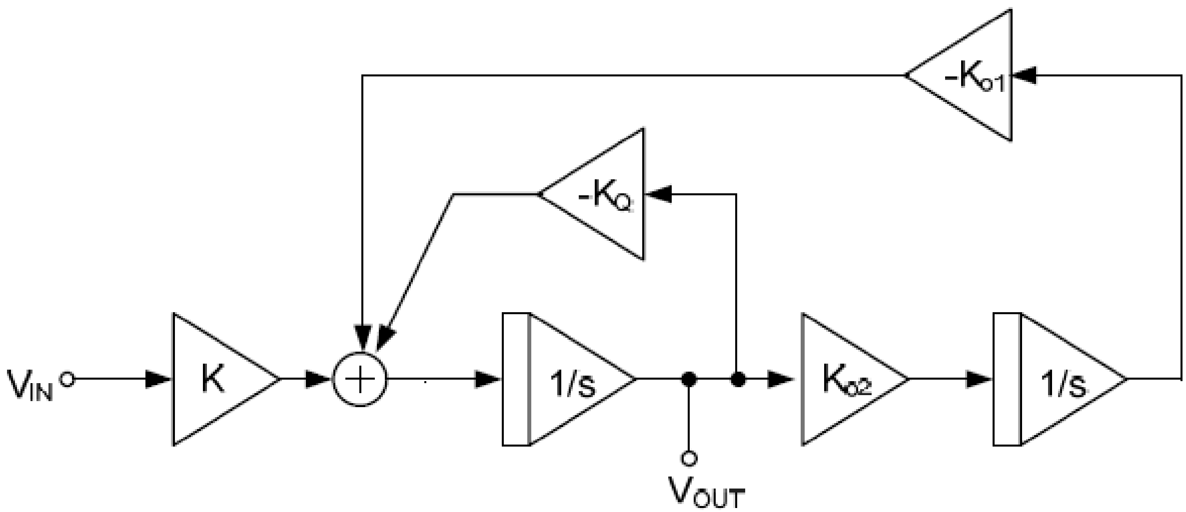


Fig. 11 Two Integrator Loop Biquad Block diagram, Family 1a.

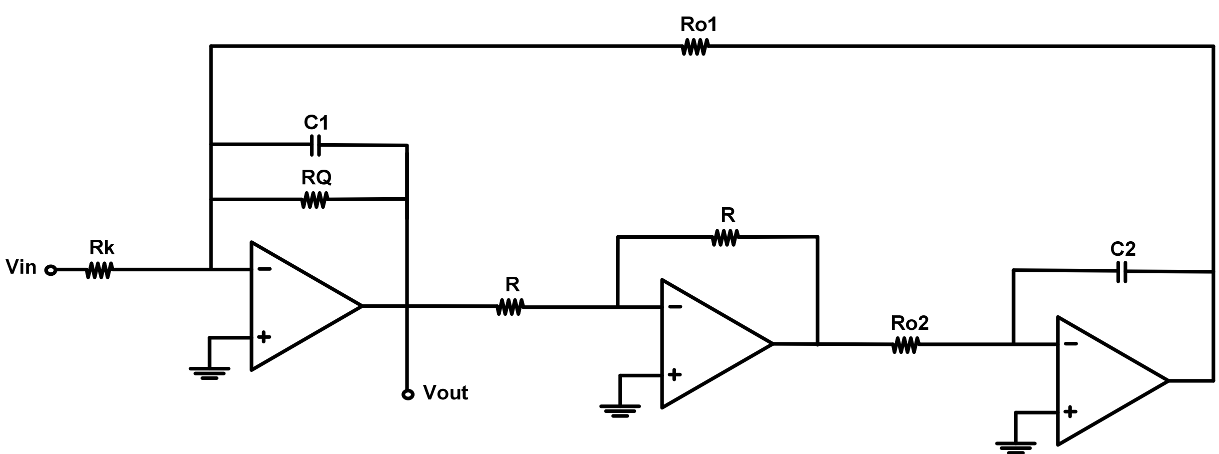


Fig. 12 Two Integrator Loop Biquad Circuit Schematic diagram.

The filter will be designed to achieve

$$\boldsymbol{w_O = 2\pi \times 5 \times 10^6} \ \&\& \ Q = 14$$

The filter Transfer function is given by:

$$\mathrm{T(s)} = \frac{s\,K}{s^2 + sK_Q + Ko1\,Ko2}$$

where

$$\mathrm{K} = \frac{1}{C_1 R_K}$$

$$K_Q = \frac{1}{C_1 R_Q}$$

$$K_{O1} = \frac{1}{C_1 R_{O1}}$$

$$K_{O2} = \frac{1}{C_2 R_{O2}}$$

$$K_Q = \frac{w_O}{Q} = \frac{2\pi \times 5 \times 10^6}{14} = 2.244 \times 10^6$$

Let

$$C_1 = 0.01 \text{ nf}$$

Then,

$$R_Q = 44.56 \text{ K}\,\Omega$$

Also,

$$K_{O1}K_{O2} = \frac{1}{C_1 C_2 R_{O1} R_{O2}} = w_0^2 = \left(2\pi \times 5 \times 10^6\right)^2$$

Let

$$K_{O1} = K_{O2} = w_O$$

Then,

$$C_2 = C_1 = 0.01 \text{ nf}$$

$$R_{O1} = R_{O2} = \sqrt{1.0132 \times 10^6}$$

Thus, their values can be

$$R_{O1} = R_{O2} = 3.183 \text{ K}\Omega$$

K is merely a magnitude scale factor, let

$$K = w_O = 2\pi \times 5 \times 10^6 \qquad \text{rad/sec}$$

<u>So that the gain of the filter at the center frequency w_O is equal to Q</u>

Then,

$$R_K = \frac{1}{C_1 K} = 3.183 \qquad \text{K}\Omega$$

Since Q > 5, then the filter bandwidth is given by:

$$\text{BW} = \frac{w_O}{Q}$$

For a band pass filter (BPF), the op-amps need to have a gain-bandwidth product (GBW) sufficient to maintain the proper operation of the filter till the second cut-off frequency

$$w_{p2} = w_{O+}\frac{w_O}{2Q} = w_O \times \frac{29}{28} = 3.2538 \times 10^7 \qquad \text{rad/sec}$$

The filter has three basic blocks: The lossy integrator, the inverter, and the lossless integrator. IF a single op-amp implementation/type/commercial product will be used in all the blocks, then the limitation on the op-amp GBW is determined by the highest requirement of the three blocks.

For the inverting amplifier stage, the gain of the circuit is given by:

$$H0(s) = \frac{\mathrm{Vo(s)}}{\mathrm{Vi(s)}} = \frac{-\mathrm{R/R}}{1 + \frac{1}{\mathrm{A(s)}}(1 + \frac{\mathrm{R}}{\mathrm{R}})} \qquad \mathrm{V/V}$$

where A(s) is the gain of the op-amp and can be approximated by $A(s) = \frac{A0 \times wp}{s}$; *GBW, A_0, w_p* are the op-amp gain bandwidth product, DC gain and 3-dB pole, respectively.

Thus, the gain of the amplifier circuit can be re-written as:

$$H0(s) = \frac{\mathrm{Vo(s)}}{\mathrm{Vi(s)}} = \frac{-1}{1 + \frac{\mathrm{s}}{\mathrm{GBW}}(2)} \qquad \mathrm{V/V}$$

Hence, the bandwidth of the amplifier is given by:

$$w3dB = \frac{\mathrm{GBW}}{2} \qquad \mathrm{rad/sec}$$

Thus,

$$\mathbf{GBW \geq 2}\boldsymbol{w_{p2}}$$

For the lossy integrator, the gain from the input terminal (Vin) to the lossy filter output (Vout) is given by:

$$\mathrm{H1(s)} = \frac{-\frac{R_Q}{R_K}}{1 + \mathrm{s}C_1R_Q + \frac{\mathrm{s}}{\mathrm{GBW}}\left(1 + \mathrm{s}C_1R_Q + \frac{R_Q}{R_K}\right)} \qquad \mathrm{V/V}$$

$$= \frac{-\mathrm{GBW}\frac{R_Q}{R_K}}{\mathrm{s}^2C_1R_Q + \mathrm{s}(\mathrm{GBW}\ C_1R_Q + 1 + \frac{R_Q}{R_K}) + \mathrm{GBW}} \qquad \mathrm{V/V}$$

Let $GBW \times C_1R_Q \gg \frac{R_Q}{R_K}$

$$GBW \gg \frac{\frac{R_Q}{R_K}}{C_1R_Q}$$

$$GBW \gg \frac{1}{C_1 R_K}$$

$$\boldsymbol{GBW \gg \mathbf{K}}$$

$$\mathrm{H1(s)} \approx \frac{-\mathrm{GBW}\frac{R_Q}{R_K}}{\mathrm{s}^2 C_1 R_Q + \mathrm{s}(\mathrm{GBW}\ C_1 R_Q + 1) + \mathrm{GBW}} \qquad \mathrm{V/V}$$

$$H1(s) \approx \frac{-\frac{R_Q}{R_K}}{(1 + \mathrm{s}C_1 R_Q)(1 + \frac{\mathrm{s}}{\mathrm{GBW}})} \qquad \mathrm{V/V}$$

Thus, to have the lossy integrator dominant pole at $1/C_1 R_Q$

$$\boldsymbol{GBW \gg \frac{1}{C_1 R_Q}}$$

$$\boldsymbol{GBW \gg K_Q}$$

Similarly, for the lossy integrator input coming from the output of the lossless integrator

To have,

$$H2(s) \approx \frac{-\frac{R_Q}{R_{O1}}}{(1 + \mathrm{s}C_1 R_Q)(1 + \frac{\mathrm{s}}{\mathrm{GBW}})} \qquad \mathrm{V/V}$$

Then, op-amp GBW should satisfy

$$\boldsymbol{GBW \gg \frac{1}{C_1 R_{O1}}}$$

$$\boldsymbol{GBW \gg K_{O1}}$$

For the lossless integrator, the integrator transfer function in case of finite GBW is given by:

$$\boldsymbol{ITF(s) \approx \frac{K_l}{s} \frac{\frac{GBW}{GBW + \sum_l |K_l|}}{\frac{s}{GBW + \sum_l |K_l|} + 1}}$$

where, $\boldsymbol{K_l}$ is the integrator gain seen from input path $\boldsymbol{l}$ fed to the integrator.

Thus, Gain error is given by

$$GE = \frac{GBW}{GBW + \sum_l |K_l|}$$

and additional pole is:

$$w_p = GBW + \sum_l |K_l|.$$

Therefore, for a lossless integrator, to maintain the lossless integrator operation over the desired frequency range, the effect of the gain error GE and extra pole w_p should be negligible.

Thus,

$$\mathbf{GBW} \gg \sum_l |K_l|$$

In case of the lossless integrator used in the filter,

$$\mathbf{GBW} \gg \boldsymbol{\frac{1}{C_2 R_{O2}}}$$

$$\boxed{\mathbf{GBW} \gg \boldsymbol{K_{O2}}}$$

Hence, the requirement on the op-amp GBW is determined by the requirement on the GBW for the lossy integrator is:

$$\mathbf{GBW} \gg \boldsymbol{max(K_Q, K, K_{O1}, K_{O2})}$$

If $\boldsymbol{K_{O1} = K_{O2} = w_O}$,

$$\mathbf{GBW} \gg \boldsymbol{max\left(\frac{w_O}{Q}, w_O\right)}$$

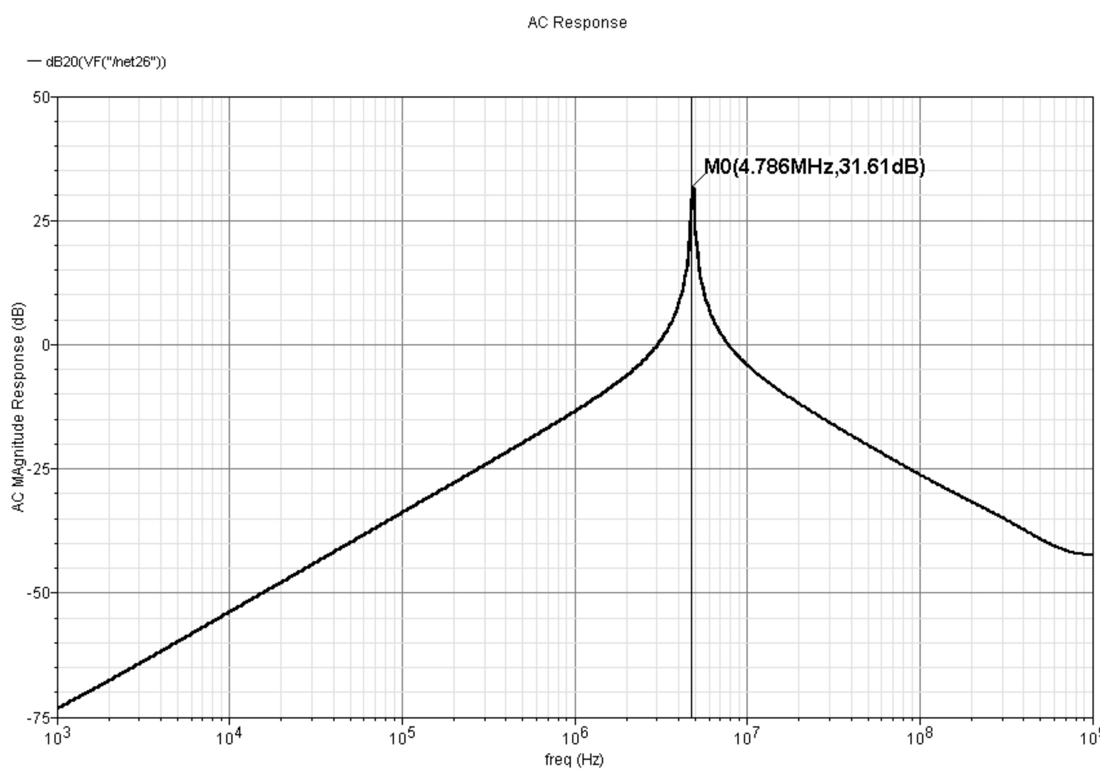
Thus,

$$\boxed{\mathbf{GBW} \gg \boldsymbol{w_O}}$$

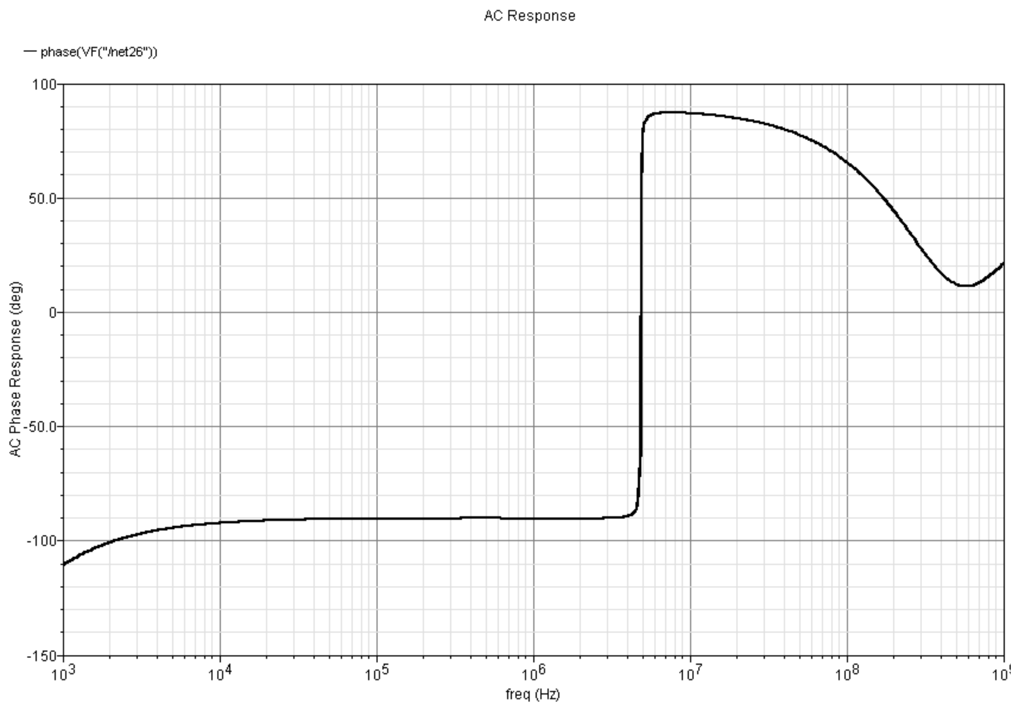
$$\textbf{GBW} \sim 10\ w_O$$

Thus, the op-amp **GBW needs to be in the order of 50 MHz**

One of the commercial op-amps that can satisfy the required design specifications is the LM7372 whose GBW is 120 MHz. Using the macro-model of the LM7372, and simulating the circuit in Cadence ADE, the following frequency domain plots for the gain of the circuit are obtained through AC analysis using a 1 V AC input signal. The resulting AC magnitude and phase responses are shown in Fig. 13.



(a)



(b)

Fig. 13 Band pass Response of Two Integrator Loop Biquad Circuit using LM7372 opamp. a) Magnitude Response. b) Phase Response.

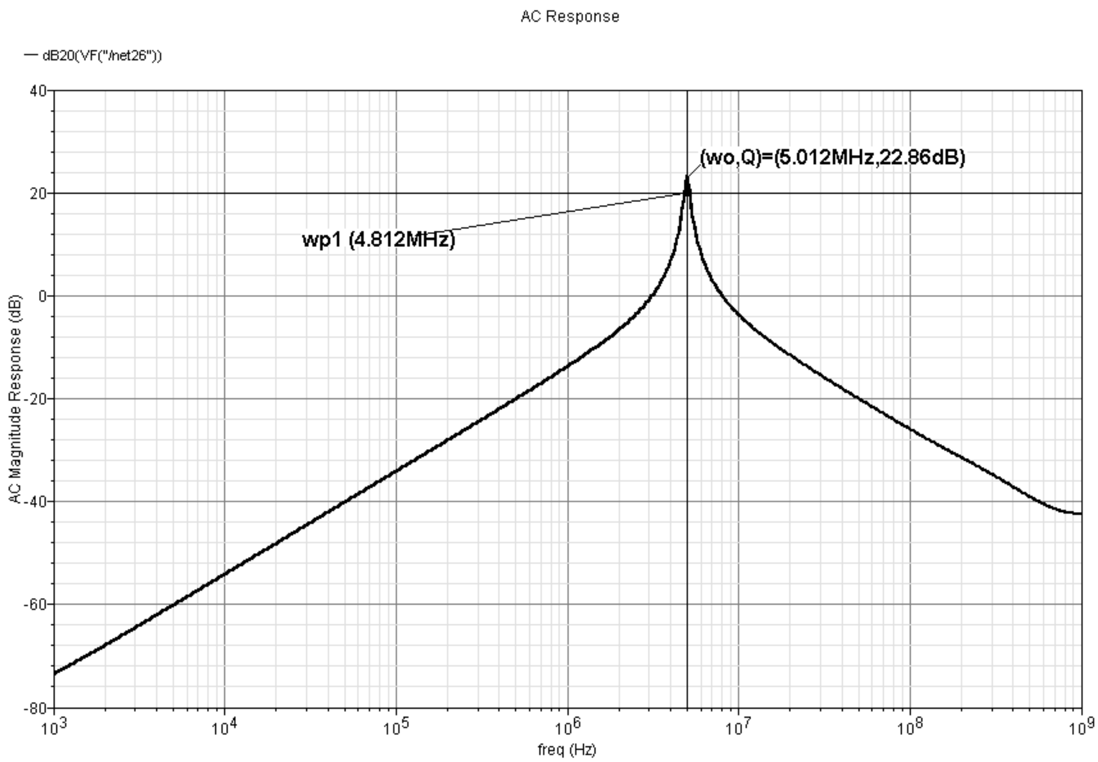
As shown from the plot in Fig. 13, the obtained Q is higher than the desired value (The gain at the center frequency is **20 Log (Q) = 31.61 dB, hence Q is 38.0627**). This is due to the Q enhancement resulting from the limited phase margin (PM) of the LM7372. Although, the LM7372 satisfied the GBW requirement, but there is another limiting factor that is coming from the finite PM.

This Q enhancement problem can be achieved by using a positive feedback compensation scheme or predistorting the frequency response of the filter (by changing the value of the resistor $\boldsymbol{R_Q}$) to achieve the desired response under the limited PM of the op-amp. Since this circuit realization is using a commercial op-amp and components are assumed to be off-chip, so the predistortion solution looks feasible because in this case, the values of the components and the amplifier specifications are known in advance so no Q-tuning loop is needed.

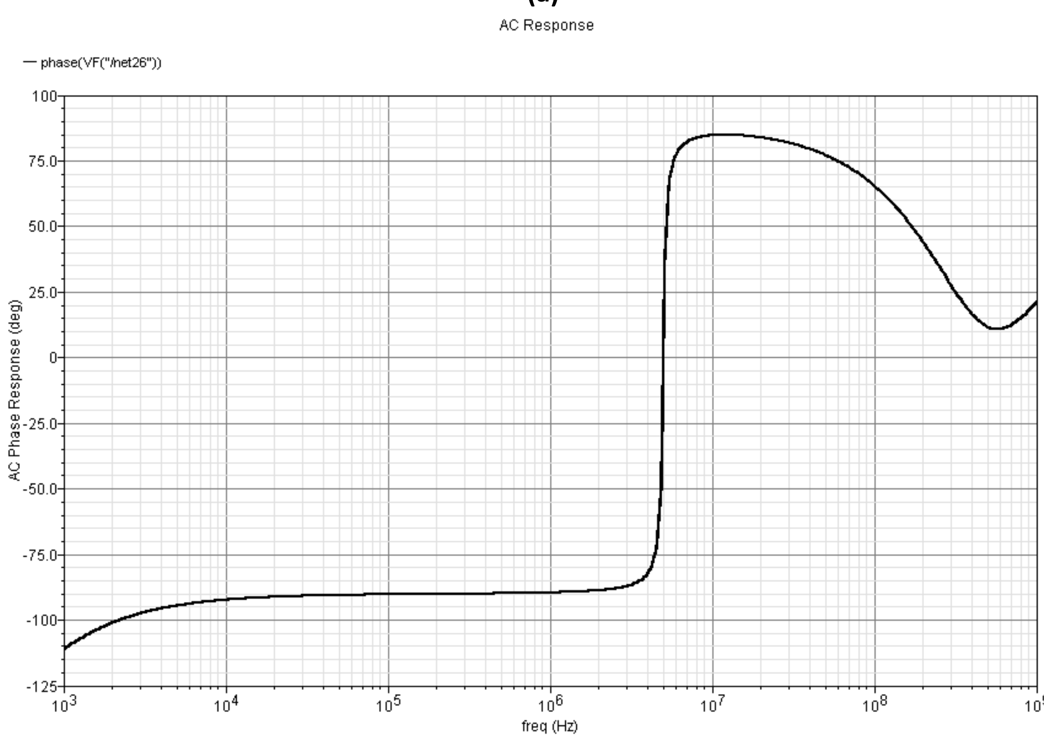
The value of the resistor R_Q has been modified (tweaked) to achieve the desired predistortion solution and their new values are given by:

$$\boldsymbol{R_Q = 210\ \mathrm{K}\,\Omega}$$

Figure 14 shows the resulting filter response using the LM7372 after modifying the value of R_Q. As can be seen in the figure, the required specifications on the Q has been achieved and the resulting peak is at 5.0119 MHz center frequency with magnitude = 22.9 dB = 20 Log(Q), thus, Q = 14.



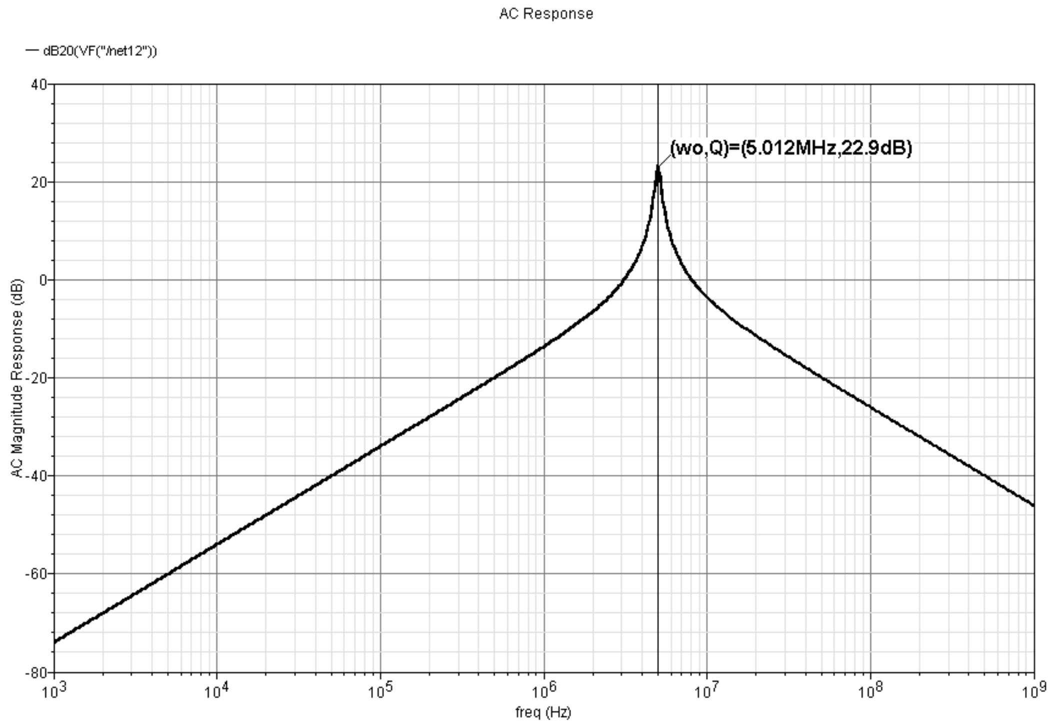
(a)



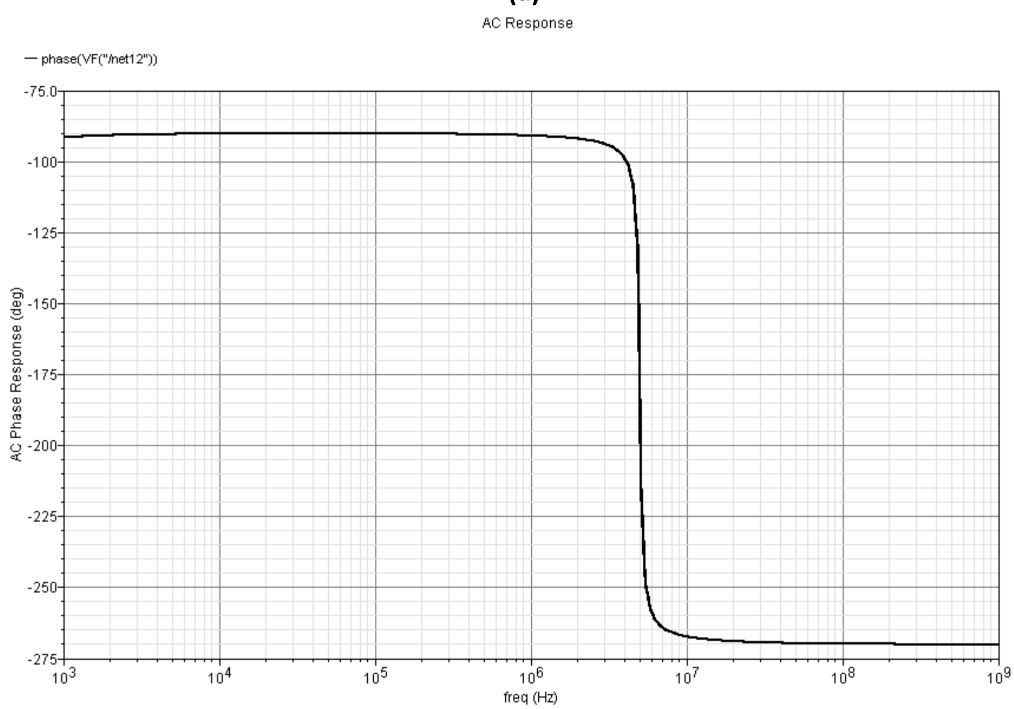
(b)

Fig. 14 Band pass AC Response of Two Integrator Loop Biquad Circuit using LM7372 opamp after modifying R_Q to compensate the Q-enhancement problem. a) Magnitude Response. b) Phase Response.

The response of an ideal op-amp with a huge GBW of 1 GHz is shown in Fig. 15. The two responses are almost the same.



(a)



(b)

Fig. 15 Band pass AC Response of Two Integrator Loop Biquad Circuit using an ideal opamp with GBW 1 GHz. a) Magnitude Response. b) Phase Response.

Conclusion:

Thus, the required specification on the Q has been achieved using a commercial op-amp of GBW of 120 MHz after predistorting the response by modifying the value of R_Q to account for the limited PM of the commercial op-amp (only 70 º).