



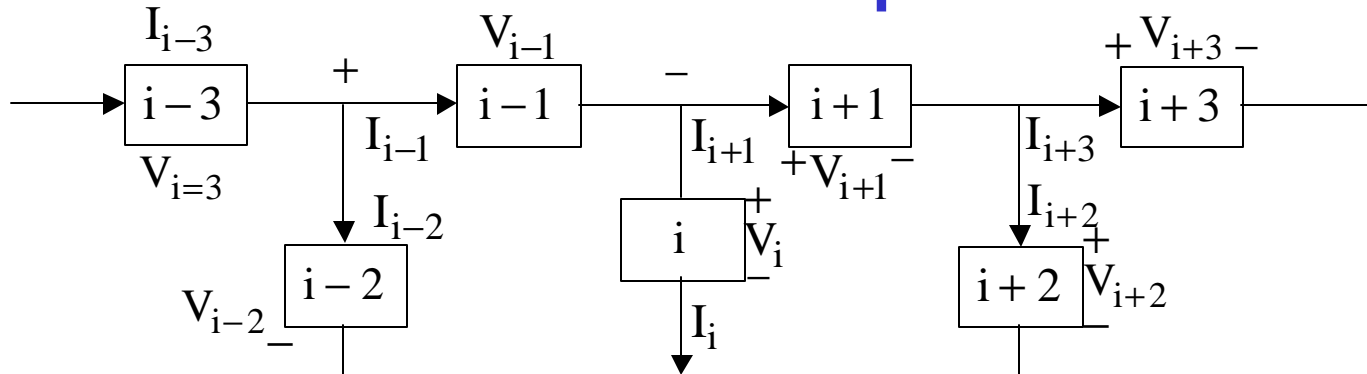
LEAP FROG

TECHNIQUE

Operational Simulation of LC Ladder Filters

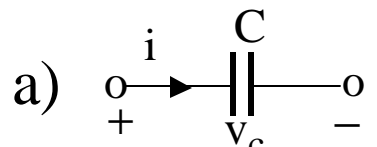
- RLC prototype \implies low sensitivity
- One form of this technique is called “Leapfrog Technique”
- Fundamental Building Blocks are
 - Integrators
 - Second-order Realizations
- Filters considered
 - LP
 - BP
 - HP
 - BE
 - $j\omega$ Zeros

General Principles

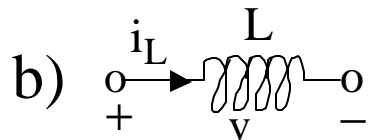


Interior Portion of a General LC Ladder Network

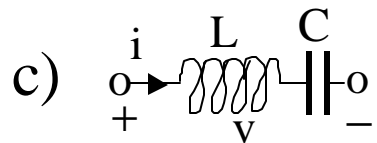
- Interior components are reactive elements only.
- $R_s \neq 0$ or $R_s = 0$ and $R_L \neq 0$ (finite)
- Branch voltages are voltage and current
- How to select the proper “**STATE**” variable ?



$$\frac{dv_c}{dt} = \frac{1}{C}i \Rightarrow v_c = \frac{1}{C} \int i(t)dt$$

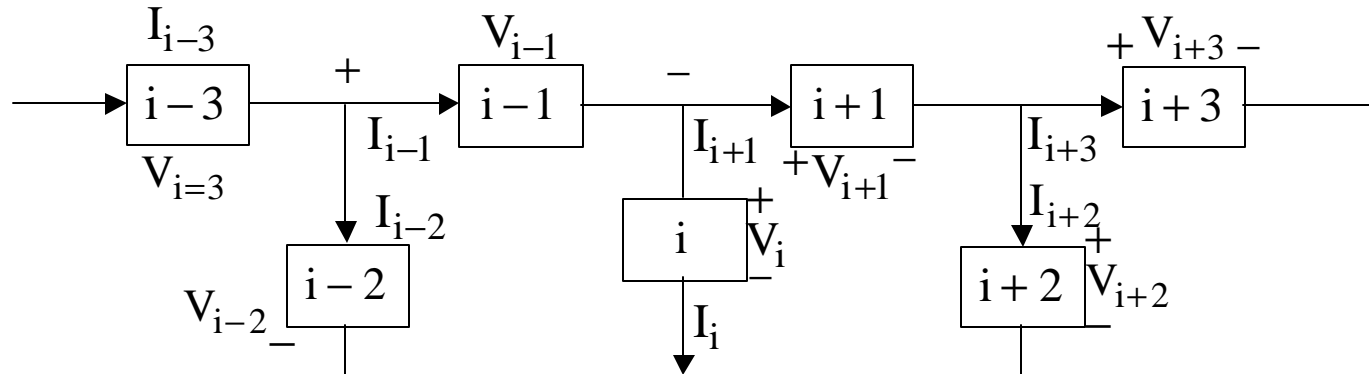


$$\frac{di_L}{dt} = \frac{1}{L}v \Rightarrow i_L = \frac{1}{L} \int v(t)dt$$



$$\frac{dv}{dt} = L \frac{d^2i(t)}{dt^2} + \frac{i}{C} \Rightarrow i = \frac{-1}{LC} \int \int i(t)dt^2 + \frac{1}{L} \int v(t)dt$$

Systematic approach by writing voltage and current equations



$$V_{i-2} = \frac{1}{Y_{i-2}} [I_{i-3} - I_{i-1}]$$

$$I_{i-1} = \frac{1}{Z_{i-1}} [V_{i-2} - V_i]$$

$$V_i = \frac{1}{Y_i} [I_{i-1} - I_{i+1}]$$

$$I_{i+1} = \frac{1}{Z_{i+1}} [V_i - V_{i+2}]$$

$$V_{i+2} = \frac{1}{Y_{i+2}} [I_{i+1} - I_{i+3}]$$

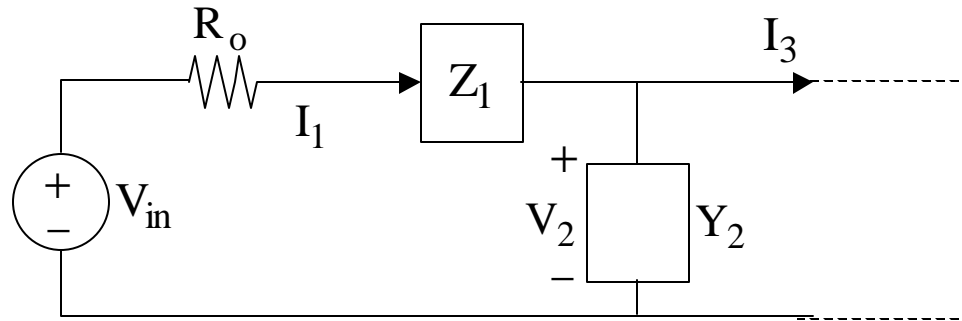
- **Immittance Functions**
- Voltage Transfer Functions, convert I to V functions

i.e., $V'_k = I_k R$

Terminations of LC Ladder Filters

Source

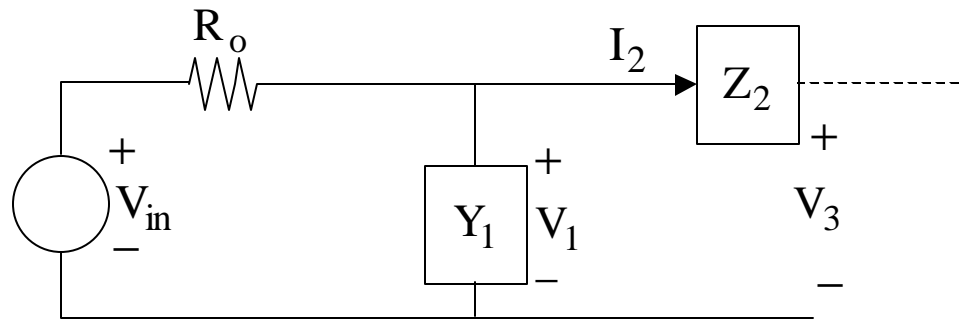
a)



$$V_1' = I_1 R = \frac{R}{Z_1} [V_{in} - \frac{R_o}{R} V_1' - V_2]$$

$$V_2 = \frac{1}{R Y_2} [V_1' - V_3]$$

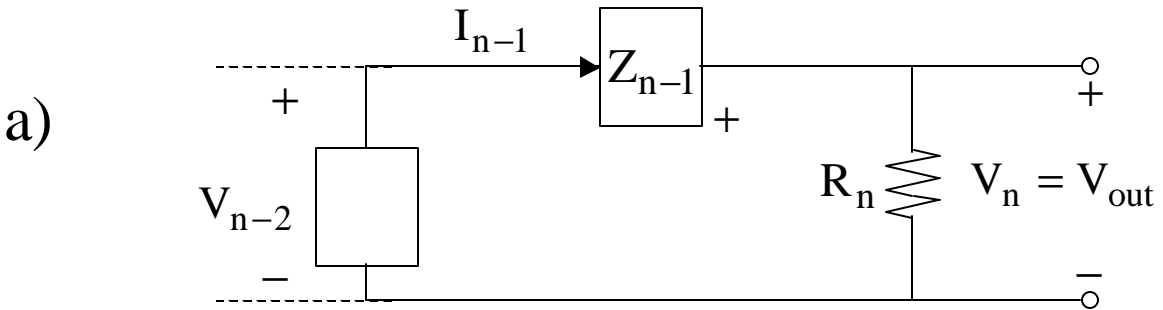
b)



$$Y_1 V_1 = \frac{V_{in} - V_1}{R_o} - I_2 \Rightarrow V_1 = \frac{1}{R Y_1} \left[\frac{R}{R_o} (V_{in} - V_1) - V_2' \right]$$

$$V_2' = \frac{R}{Z_2} [V_1 - V_3]$$

Load termination



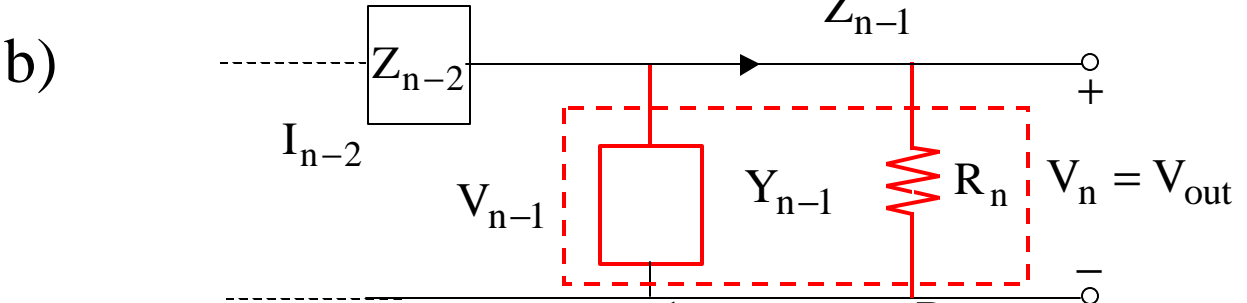
$$V_{out} = V_n = \frac{R_n}{R} V'_{n-1}$$

or $V_{out} = R_n [(V_{n-2} - V_{out}) / Z_{n-1}]$

$$V_{out} = \frac{R}{Z_{n-1}} \frac{R_n}{R} (V_{n-2} - V_{out})$$

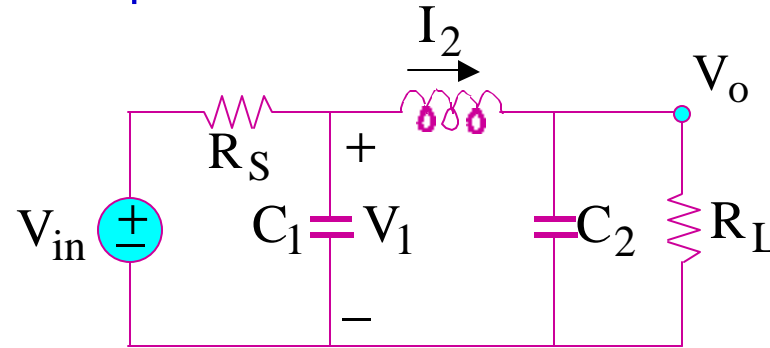
$$V_{out} = \frac{\frac{R_n}{R} V_{n-2}}{1 + \frac{R_n}{Z_{n-1}}}$$

Lossy
Integrator



$$V_{out} = \frac{1}{R Y_{n-1}} [V'_{n-2} - \frac{R}{R_n} V_{out}]$$

The approach to map a passive RLC prototype is to pick the state-variables which can be expressed as integrators, since integrators are the basic building block. By applying KCL, KVL, KCL, ..., as many times as the order of the filter, one can write the state-equations that can be implemented by active filters. an example is shown below:



A Typical Passive RLC Filter

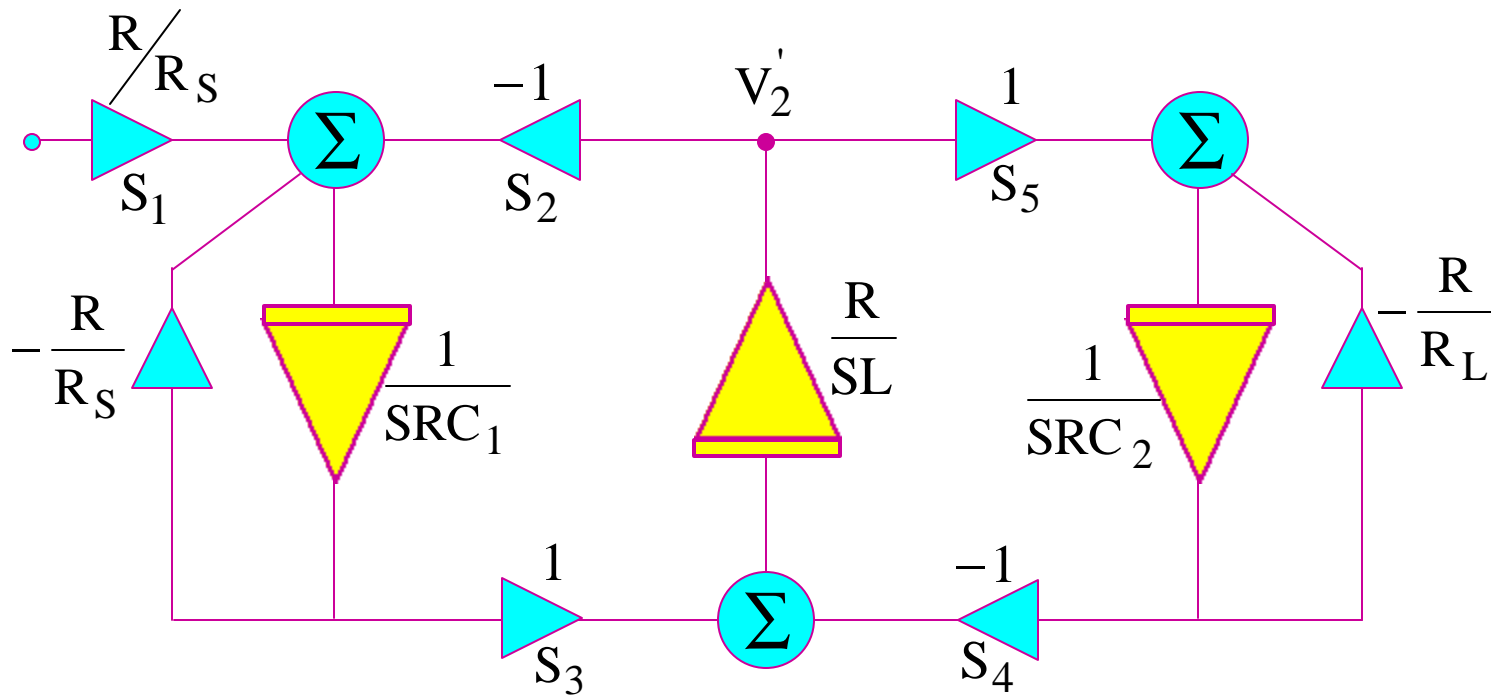
$$\text{KCL} \quad V_1 = \frac{1}{S R C_1} \left[\frac{R}{R_S} (V_{in} - V_1) - V_2' \right] \quad (15a)$$

where

$$V_2' = I_2 R, \quad R \text{ is an arbitrary value} \quad (15b)$$

$$\text{KVL} \quad V_2' = \frac{R}{S L} (V_1 - V_o) \quad (16)$$

$$\text{KCL} \quad V_o = \frac{1}{S R C_2} \left[V_2' - \frac{R}{R L} V_o \right] \quad (17)$$



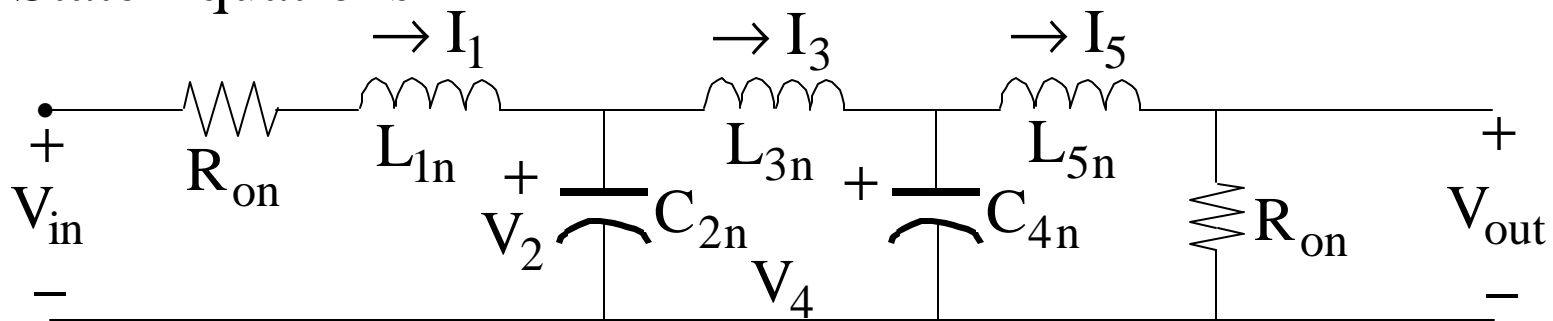
Signal Flow Graph of RLC Prototype of Fig. Shown in previous page.

Low-Pass Ladder Filters

(Zeros at Infinity) (All Poles)

Example: A Fifth-Order LP Filter

- 5 State Variables $[I_1, V_2, I_3, V_4, I_5]$
- 5 State Equations



KVL-KCL-KVL-KCL-KVL (Sequence)

*
$$V_{in} - I_1(R_{on} + sL_{1n}) - V_2 = 0$$

Current Analogs $V_1' = I_1 R$

$$V_1' = \frac{R}{sL_{in}} [V_{in} - V_2 - \frac{R_{on}}{R} V_1'] \quad (1)$$

$$I_1 - I_3 - sC_{2n} V_2 = 0$$

$$V_2 = \frac{1}{sRC_{2n}} [V_1' - V_3'] \quad (2)$$

$$V_2 - sL_{3n} I_3 - V_4 = 0$$

$$V_3' = \frac{R}{sL_{3n}} [V_2 - V_4] \quad (3)$$

$$I_3 - sC_{4n} V_4 - I_5 = 0$$

$$V_4 = \frac{1}{sRC_{4n}} [V_3' - V_5'] \quad (4)$$

$$V_4 - sL_{5n} I_5 - I_5 R_{6n} = 0$$

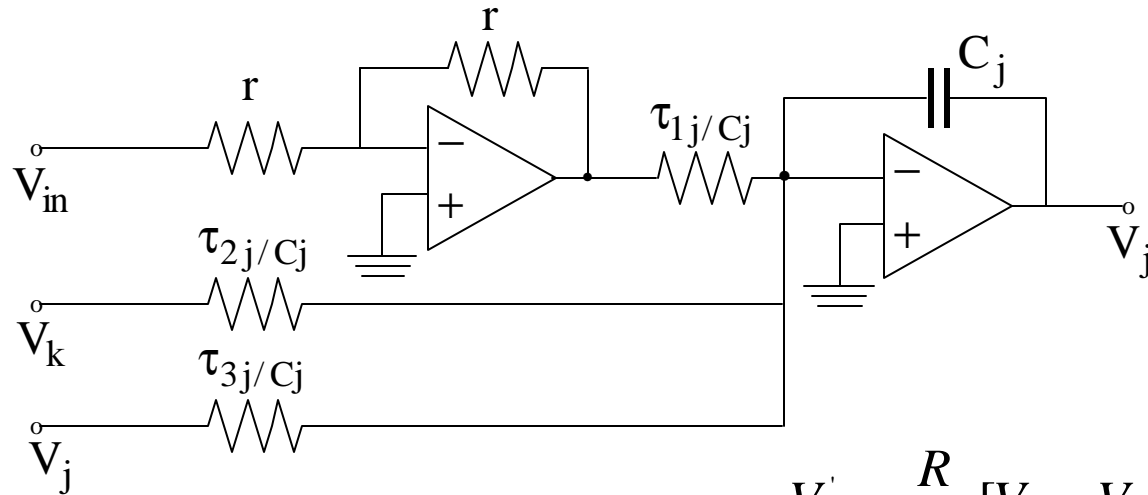
$$V_5' = \frac{R}{sL_{5n}} [V_4 - \frac{R_{6n}}{R} V_5']$$

But

$$V_{\text{out}} = I_5 R_{6n} = \frac{V_5'}{R} R_{6n}$$

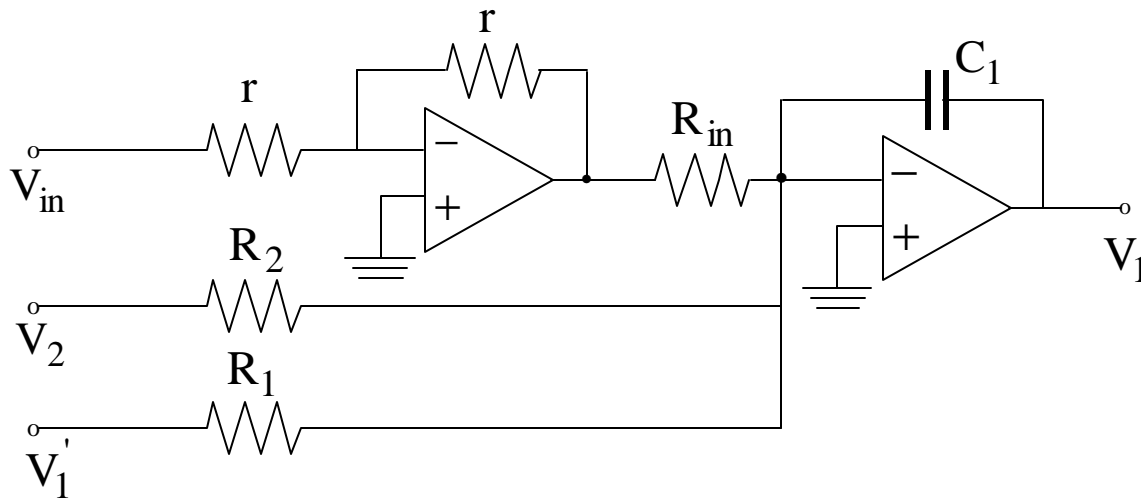
$$V_{\text{out}} = \frac{R_{6n}}{R} \frac{R}{sL_{5n}} [V_4 - \frac{R_{6n}}{R} V_{\text{out}} \frac{R}{R_{6n}}] \quad (5)$$

Active RC Building Blocks



$$V_1' = \frac{R}{sL_{in}} [V_{in} - V_2 - \frac{R_{on}}{R} V_1']$$

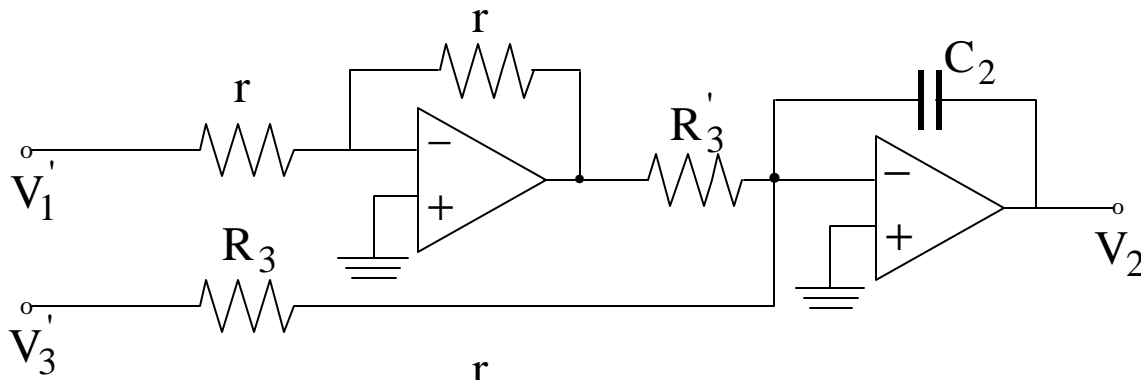
Thus, eq(1) is implemented as follows



$$R_{in} C_1 = \frac{L_{in}}{R}$$

$$R_2 C_1 = \frac{L_{in}}{R}$$

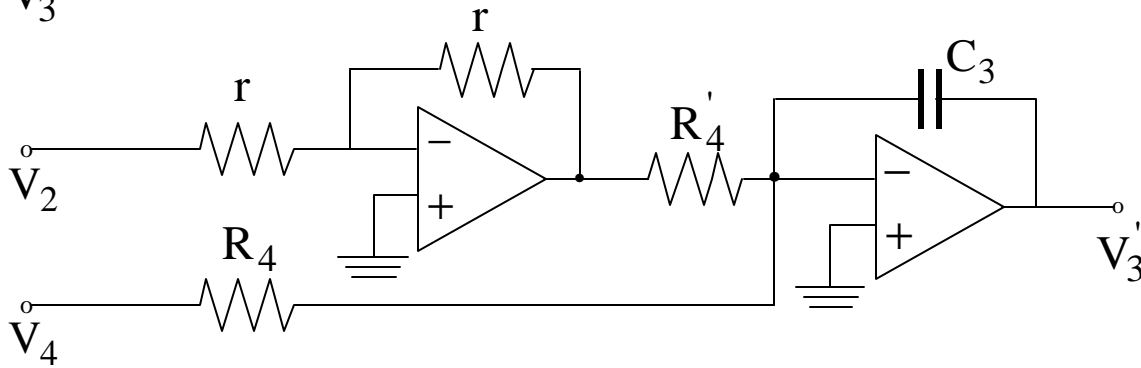
$$R_1 C_1 = \frac{L_{in}}{R_{on}}$$



$$V_2 = \frac{1}{sRC_{2n}} [V_1' - V_3']$$

$$RC_{2n} = R_3' C_2$$

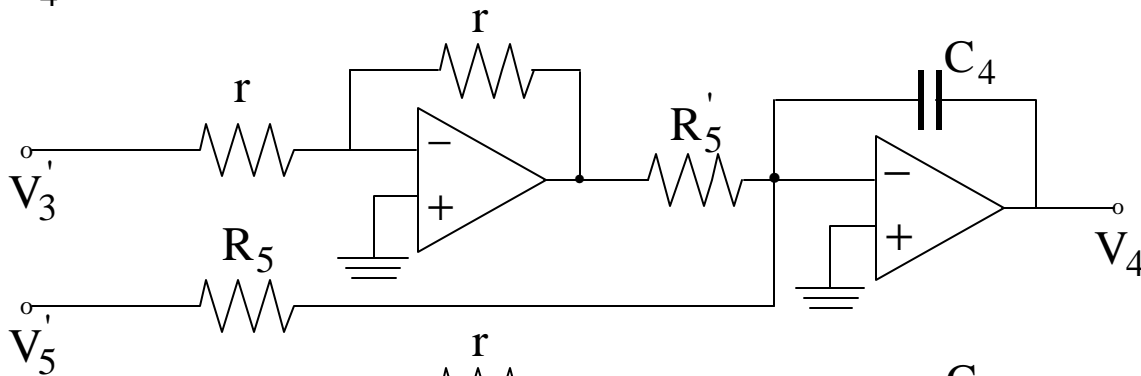
$$RC_{2n} = R_3 C_2$$



$$V_3' = \frac{R}{sL_{3n}} [V_2 - V_4]$$

$$R_4' C_3 = L_{3n} / R$$

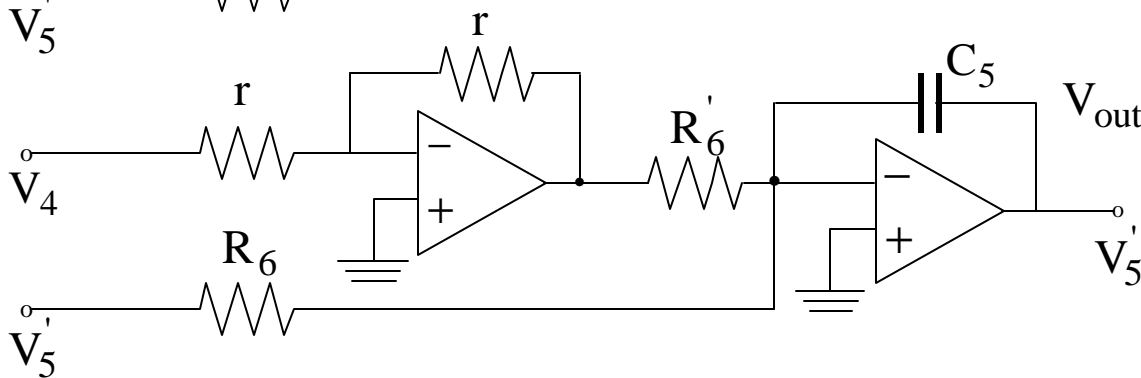
$$R_4 C_3 = L_{3n} / R$$



$$V_4 = \frac{1}{sRC_{4n}} [V_3' - V_5']$$

$$R_5' C_4 = RC_{4n}$$

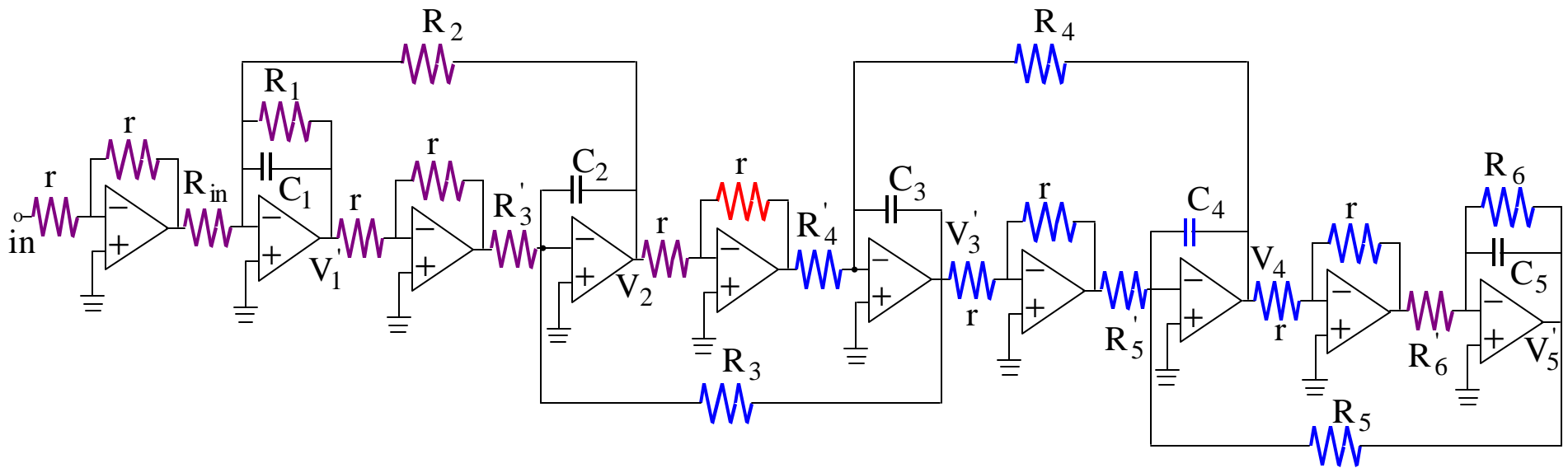
$$R_5 C_4 = RC_{4n}$$



$$V_{out} = \frac{R_{6n}}{R} \frac{R}{sL_{5n}} [V_4 - \frac{R_{6n}}{R} V_{out} \frac{R}{R_{6n}}]$$

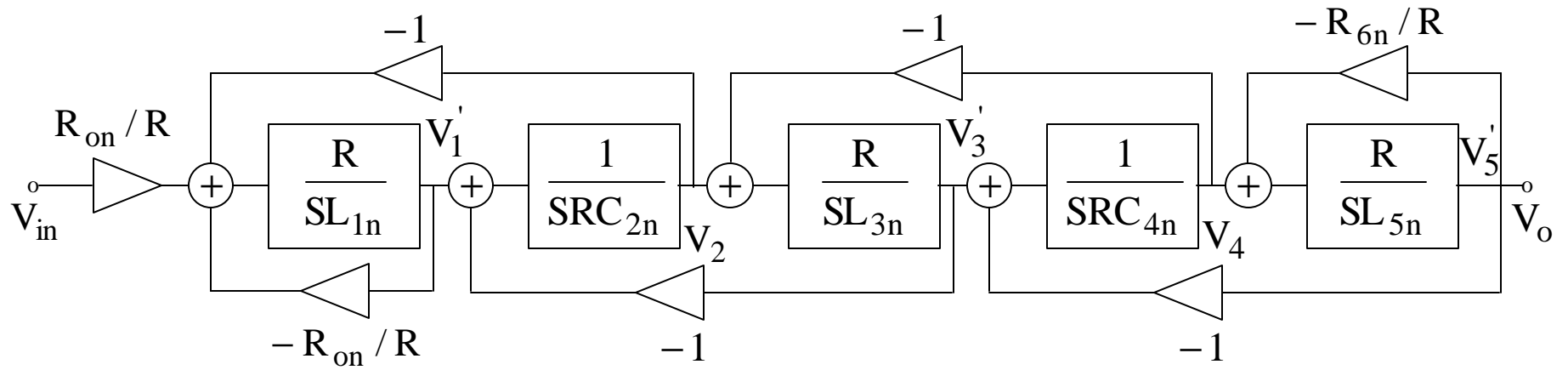
$$R_6' C_5 = L_{5n} / R_{6n}$$

$$R_6 C_5 = L_{5n} / R_{6n}$$

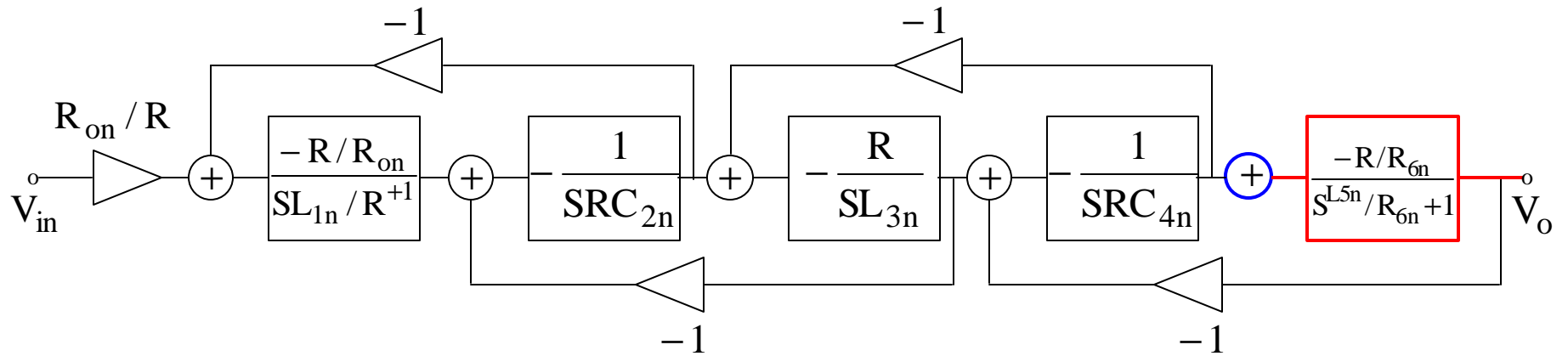


Low-Pass 5th-Order Active -RC

Leapfrog Filter



Leapfrog Building Block Representation

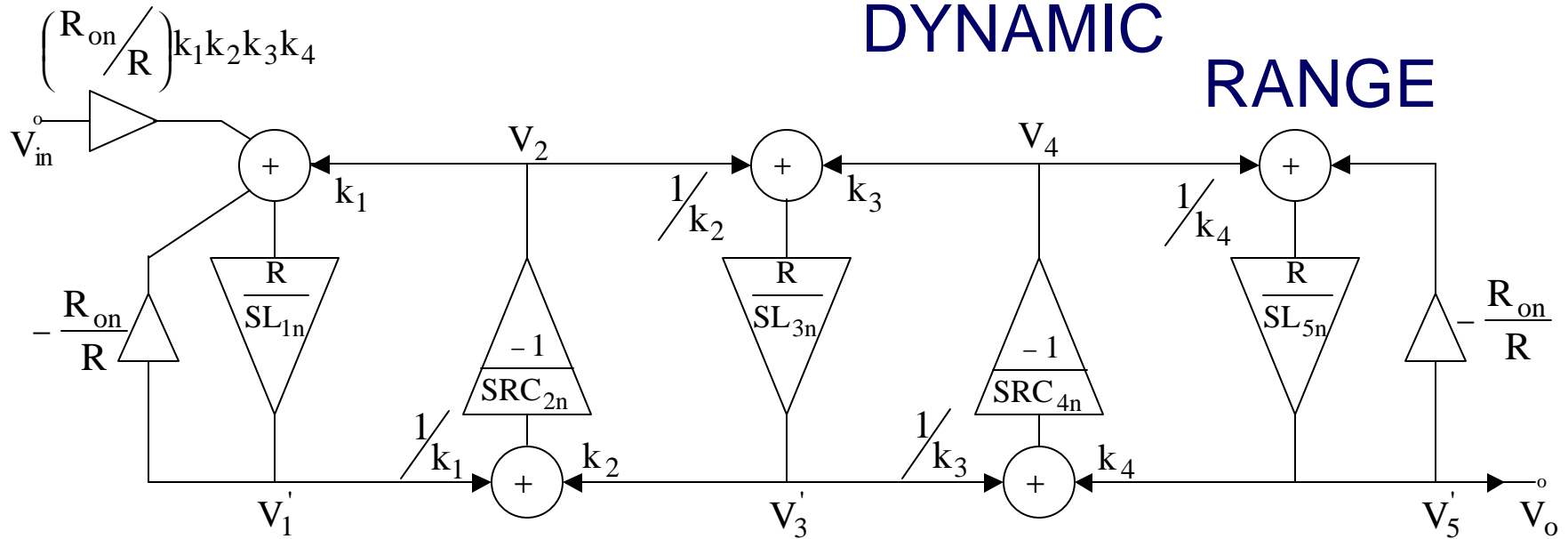


SCALING

MAXIMUM

DYNAMIC

RANGE



$$V_{in} = 1$$

$$V_4 = V_o \left(s \frac{L_{5n}}{R} + \frac{R_{6n}}{R} \right) ; \quad \frac{V_{o \max}}{V_{3 \max}} = k_3 \quad ; \quad \frac{V_{o \max}}{V_{1 \max}} = k_1$$

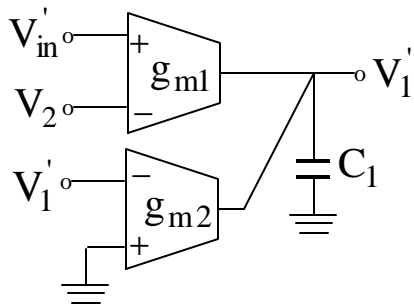
$$V_4 = T(s) \left(s \frac{L_{5n}}{R} + \frac{R_{6n}}{R} \right) ; \quad \frac{V_{o \max}}{V_{2 \max}} = k_2$$

$$\frac{V_{o \max}}{V_{4 \max}} = k_4$$

OTA-C Implementation

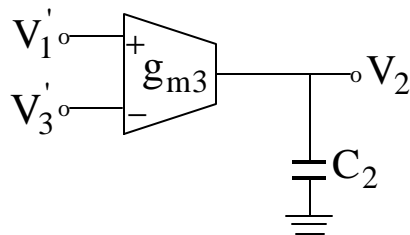
$$V_1' = \frac{R}{sL_{in}} [V_{1n} - V_2 - \frac{R_{on}}{R} V_1']$$

Thus, EQ (1) is implemented as



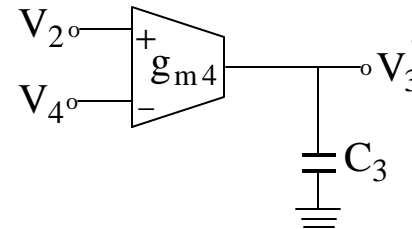
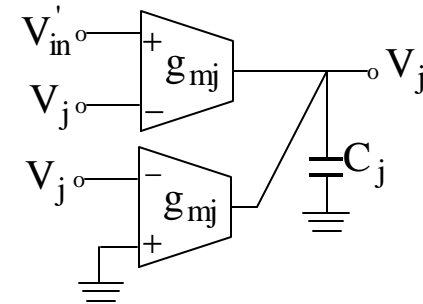
$$\frac{g_{m1}}{C_1} = \frac{L_{1n}}{R}$$

$$\frac{g_{m2}}{C_2} = \frac{L_{1n}}{R_{on}}$$



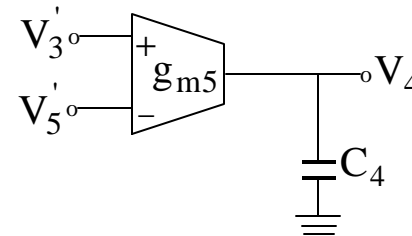
EQ (2)

$$\frac{g_{m3}}{C_2} = RC_{2n}$$



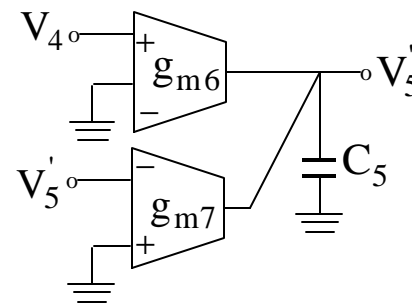
EQ (3)

$$\frac{g_{m4}}{C_3} = \frac{L_{3n}}{R}$$



EQ (4)

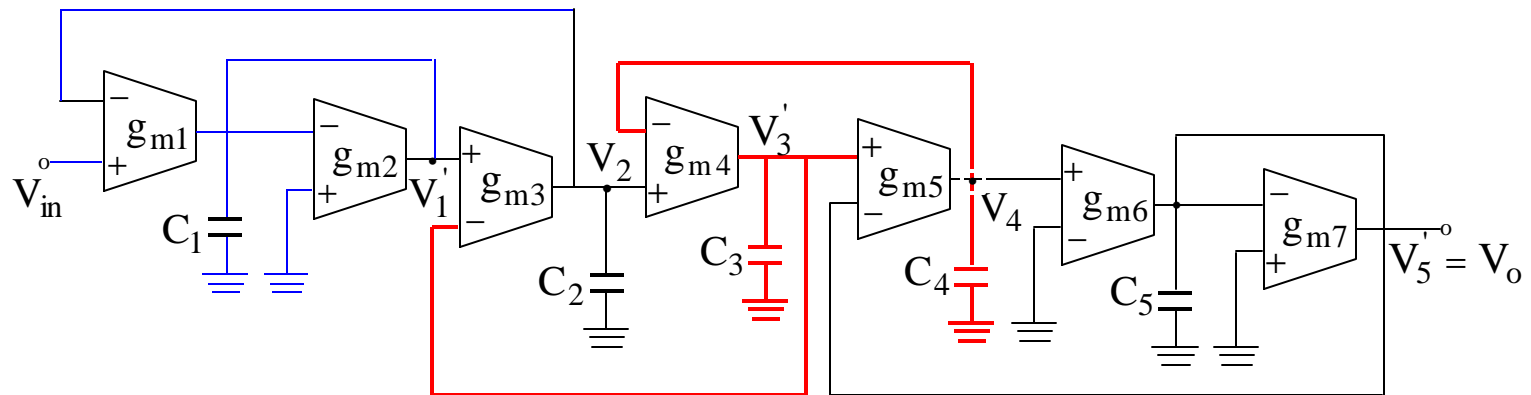
$$\frac{g_{m5}}{C_4} = RC_{4n}$$



EQ (5)

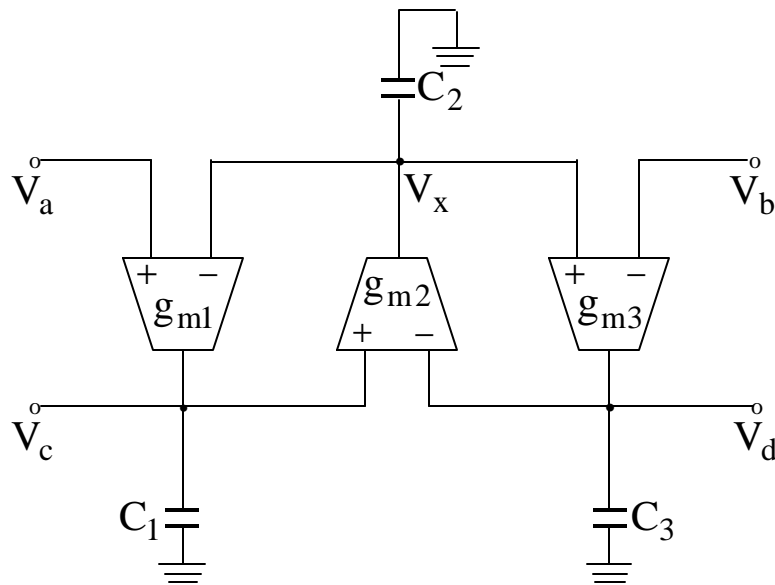
$$\frac{g_{m6}}{C_5} = \frac{L_{5n}}{R_{on}}$$

$$\frac{g_{m7}}{C_5} = \frac{L_{5n}}{R_{on}}$$

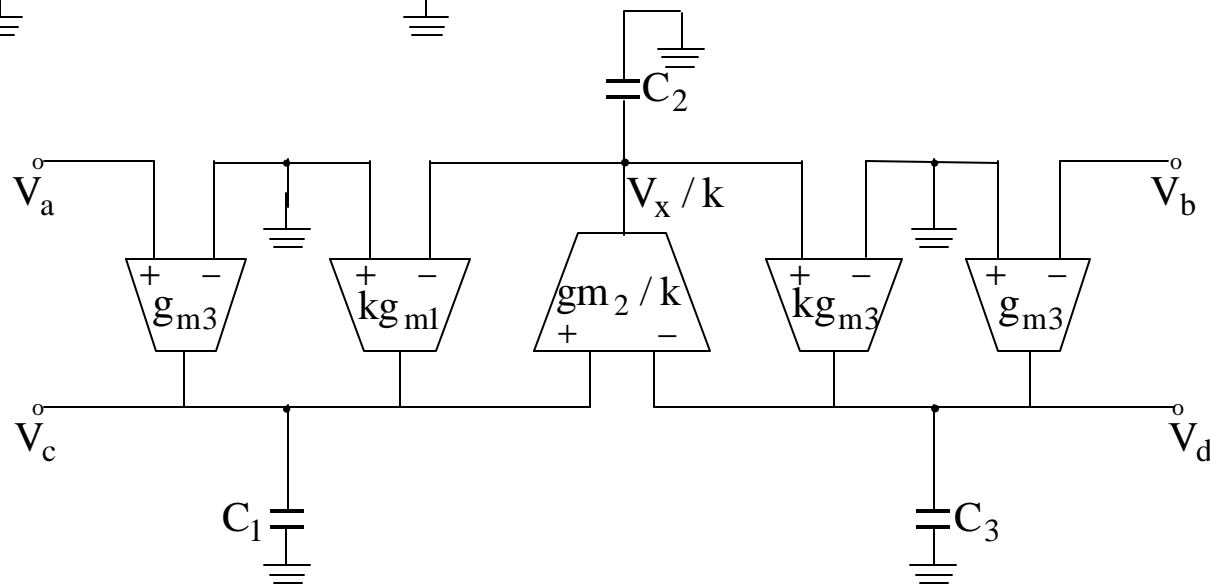


5th-Order OTA-C Leapfrog Implementation

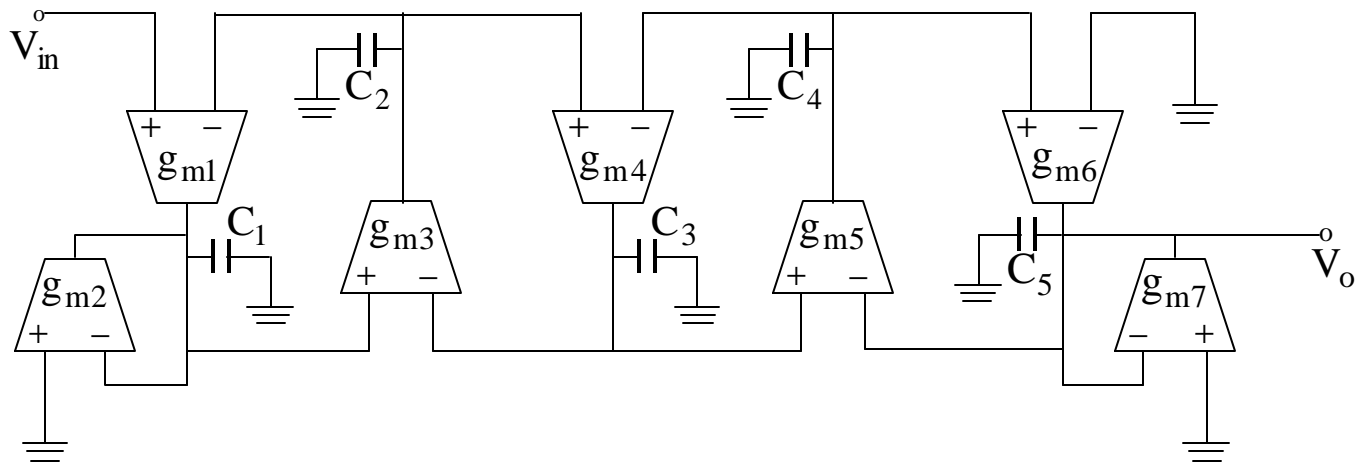
Voltage Scaling of OTA-C All Pole Leapfrog Filters.

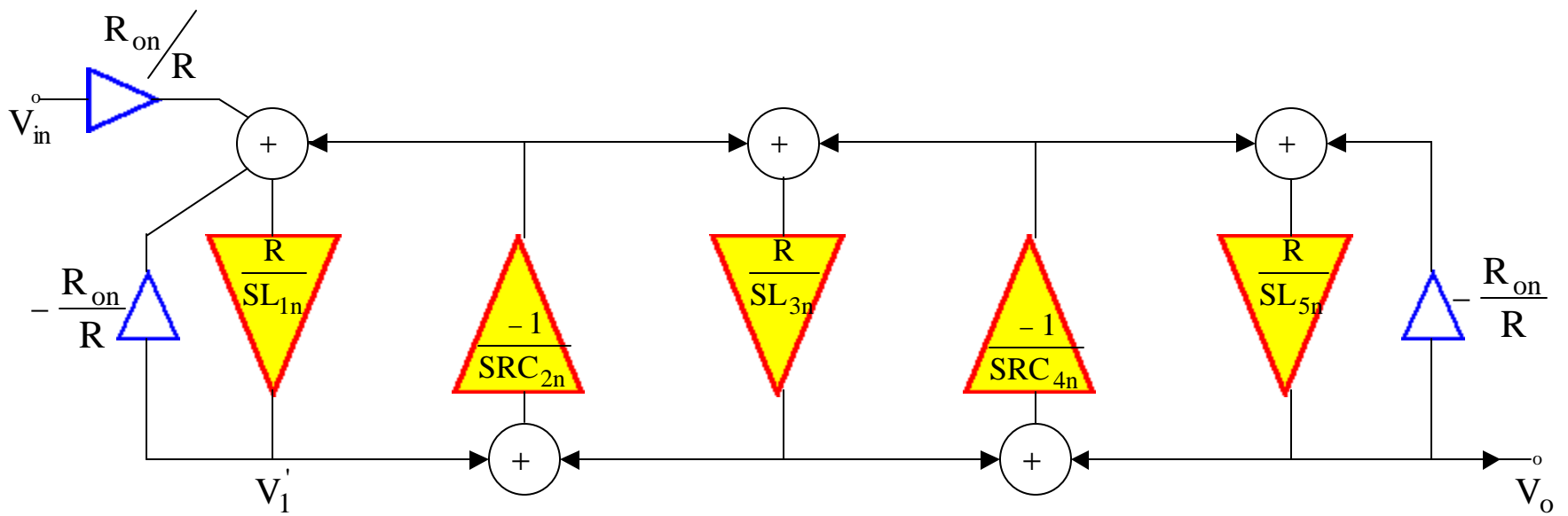


$$V_x = g_{m2}(V_c - V_d)/sC_2$$



Changing $V_x \rightarrow \frac{V_x}{k}$ without changing V_a , V_b , V_c , and V_d





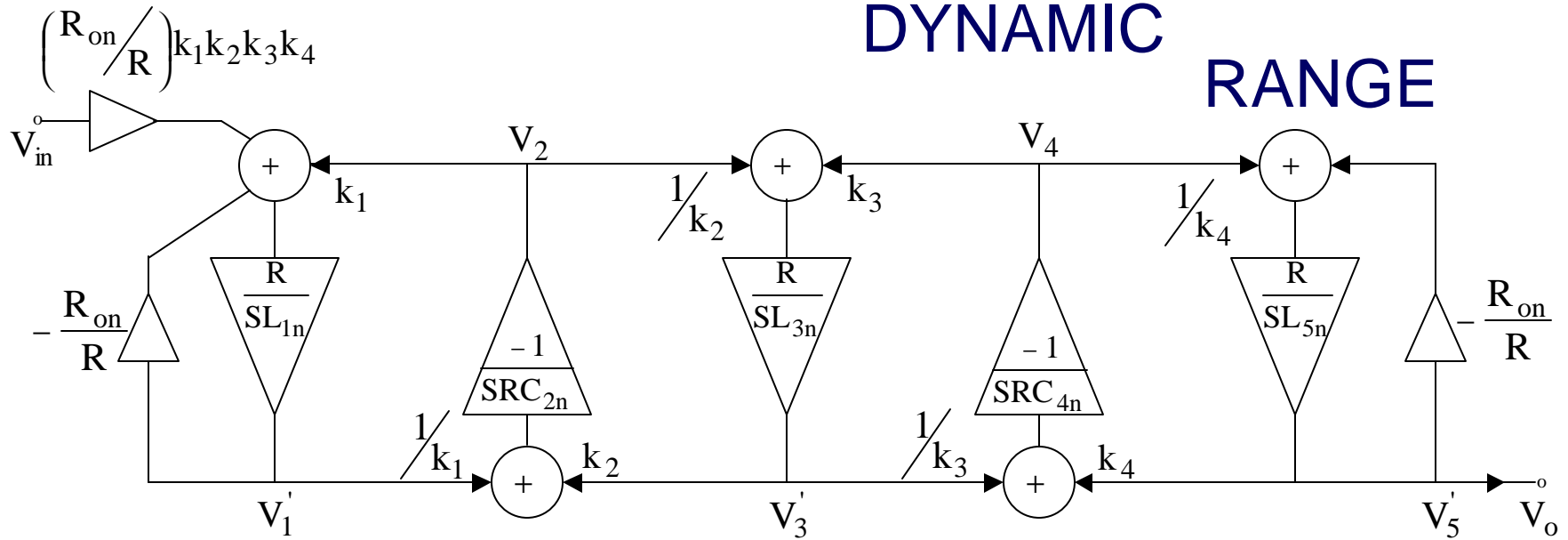
System Level Representation: 5th-Order LP Leapfrog Filter

SCALING

MAXIMUM

DYNAMIC

RANGE



$$V_{in} = 1$$

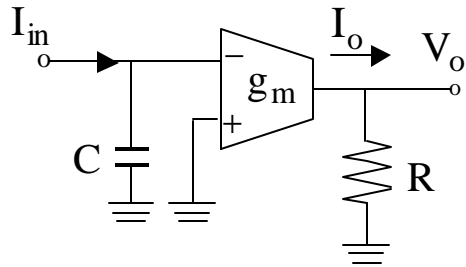
$$V_4 = V_o \left(s \frac{L_{5n}}{R} + \frac{R_{6n}}{R} \right) ; \quad \frac{V_{o \max}}{V_{3 \max}} = k_3 \quad ; \quad \frac{V_{o \max}}{V_{1 \max}} = k_1$$

$$V_4 = T(s) \left(s \frac{L_{5n}}{R} + \frac{R_{6n}}{R} \right) ; \quad \frac{V_{o \max}}{V_{2 \max}} = k_2$$

$$\frac{V_{o \max}}{V_{4 \max}} = k_4$$

Current Level Scaling Technique

Case 1



R represents input resistance of a current-mirror

$$I_o = -g_m / sC$$

$$V_o = I_o R = -\frac{g_m R}{sC}$$

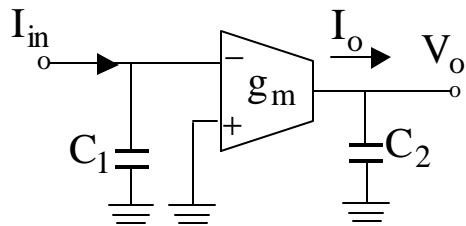
$$kI_o = -k \frac{g_m}{sC} \begin{cases} \text{option 1} & g_m \rightarrow kg_m \\ \text{option 2} & C \rightarrow C/k \end{cases}$$

$$V_o = kI_o R$$

If V_o needs to be constant, then R must be decreased by k .

In summary $\begin{cases} \text{option 1} & g_m \rightarrow kg_m & \& & R \rightarrow R/k \\ \text{option 2} & C \rightarrow C/k & \& & R \rightarrow R/k \end{cases}$

Case 2

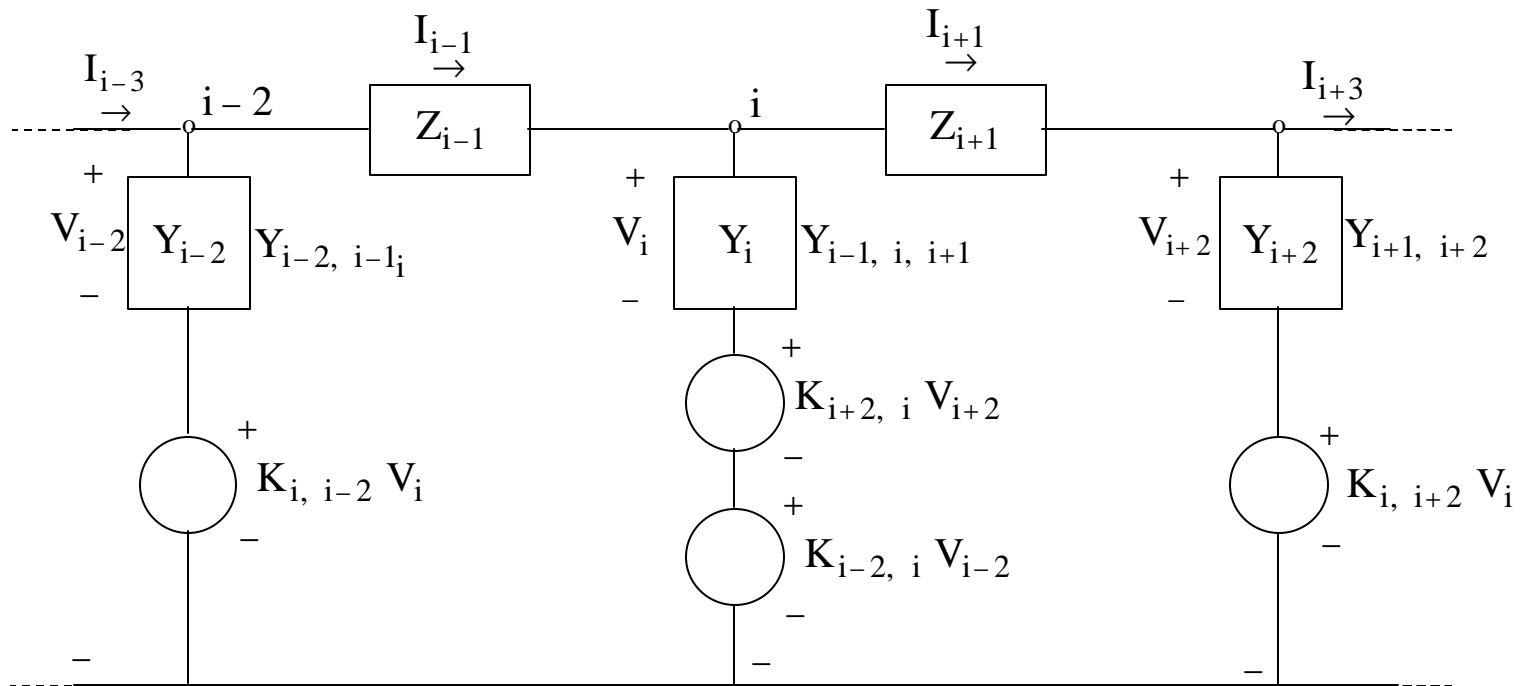
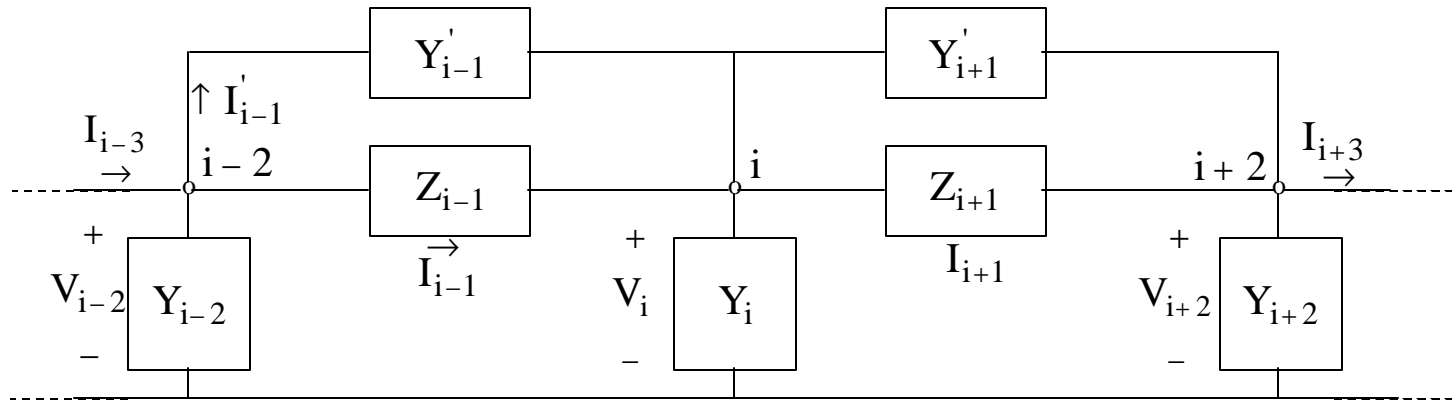


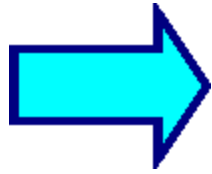
C_2 represents input impedance of next block

$$I_o = -\frac{g_m}{sC_1}$$

$$V_o = I_o \frac{1}{sC_2}$$

LC Ladder Filters with Finite Zeros





Dependent Sources to Replace the Bridging Admittances

KCL at $i-2$

$$I_{i-3} - I_{i-1} = Y_{i-2}V_{i-2} + Y'_{i-1}[V_{i-2} - V_i]$$

or

$$V_{i-2} = \left[\frac{Y'_{i-1}}{Y_{i-2} + Y'_{i-1}} \right] V_i + \frac{I_{i-3} - I_{i-1}}{Y_{i-2} + Y'_{i-1}}$$

$$V_{i-2} = K_{i,i-2}V_i + \frac{(I_{i-3} - I_{i-1})}{Y_{i-2,i-1}}$$

KCL @ NODE i

$$I_{i-1} - I_{i+1} = Y_i V_i - Y'_{i-1}(V_{i-2} - V_i) + Y'_{i+1}(V_i - V_{i+2})$$

or

$$V_i = \frac{Y'_{i-1}}{Y'_{i-1} + Y_i + Y'_{i+1}} V_{i-2} + \frac{Y'_{i+1}}{Y'_{i-1} + Y_i + Y'_{i+1}} V_{i+2} + \frac{I_{i+1} - I_{i-1}}{Y'_{i-1} + Y_i + Y'_{i+1}}$$

$$V_i = K_{i-2,i}V_{i-2} + K_{i+2,i}V_{i+2} + \frac{V'_{i+1} - V'_{i-1}}{RY_{i-1,i,i+1}}$$

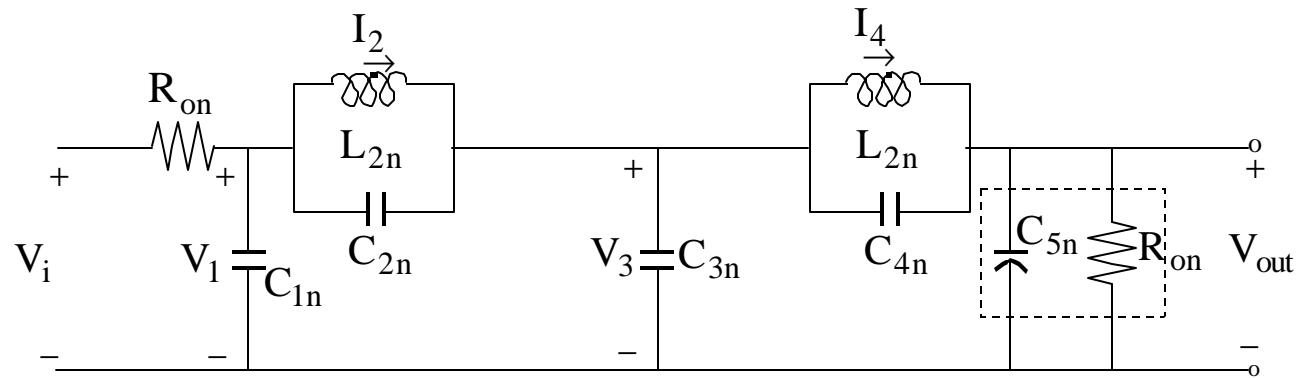
In a similar for KCL for node $i+2$ and solving for V_{i+2} results

$$V_{i+2} = K_{i,i+2}V_i + (V'_{i+1} - V'_{i+3})/RY'_{i,i+2}$$

LC Ladder Simulation

1. Start from the desired LC LP prototype network satisfying SPECS.
2. Eliminate shunt elements in the series branches or series element in the shunt branches.
3. Normalize and transform the network if the realization is not low-pass.
4. Select the state variables such that $\frac{dx}{dt} = f(x, \mu s)$
5. Solve for each state variable in terms of the other states and output variable including the proper scaling.
6. Synthesize each equation with its corresponding building block.

Example: Low-Pass With Finite $j\omega$ Zeros



$$C_{12n} = C_{1n} + C_{2n}$$

$$C_{234n} = C_{2n} + C_{3n} + C_{4n}$$

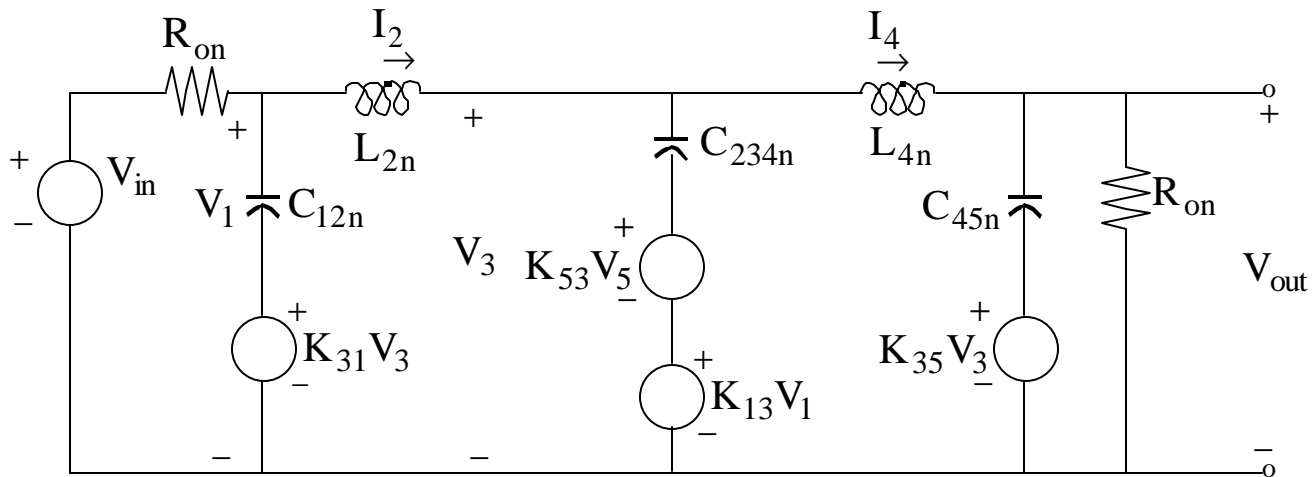
$$C_{45n} = C_{4n} + C_{5n}$$

$$K_{31} = \frac{C_{2n}}{C_{12n}}$$

$$K_{53} = \frac{C_{4n}}{C_{234n}}$$

$$K_{13} = \frac{C_{2n}}{C_{234n}}$$

$$K_{35} = \frac{C_{4n}}{C_{45n}}$$



$$C_{12n} = C_{1n} + C_{2n}$$

$$C_{234n} = C_{2n} + C_{3n} + C_{4n}$$

$$C_{45n} = C_{4n} + C_{5n}$$

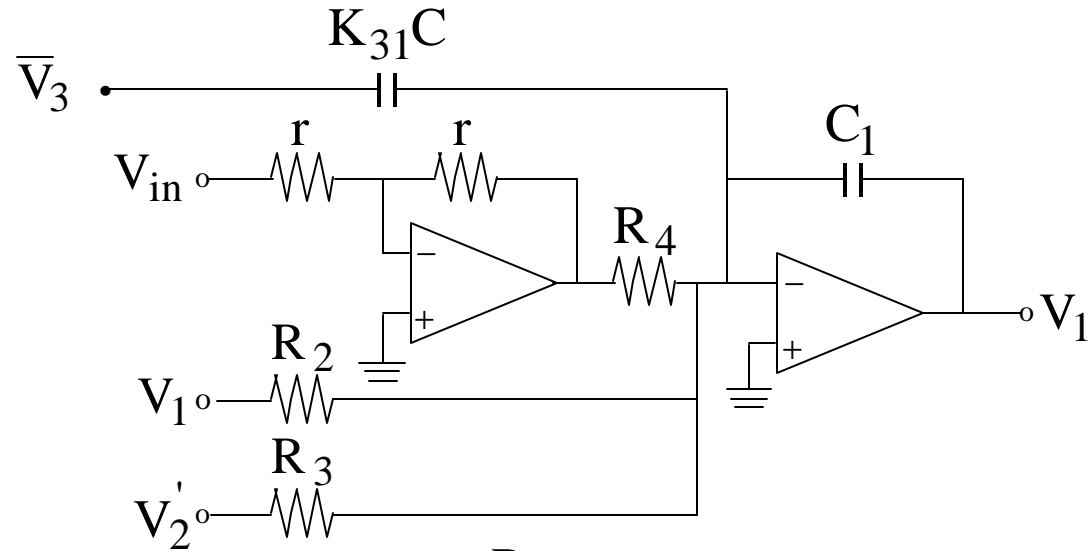
$$K_{31} = \frac{C_{2n}}{C_{12n}}$$

$$K_{53} = \frac{C_{4n}}{C_{234n}}$$

$$K_{13} = \frac{C_{2n}}{C_{234n}}$$

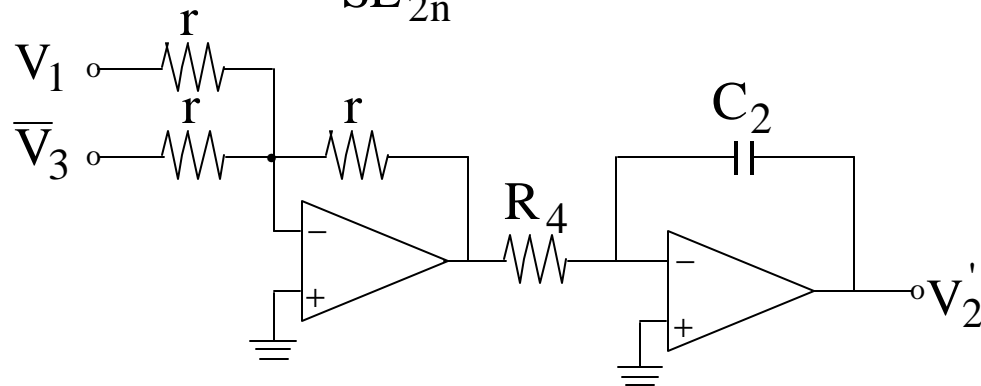
$$K_{35} = \frac{C_{4n}}{C_{45n}}$$

$$V_1 = \frac{1}{SC_{12n}} \left[\frac{V_{in}}{R_{on}} - \frac{V_1}{R_{on}} - \frac{V_2'}{R} \right] + K_{31} V_3 \quad (1)$$



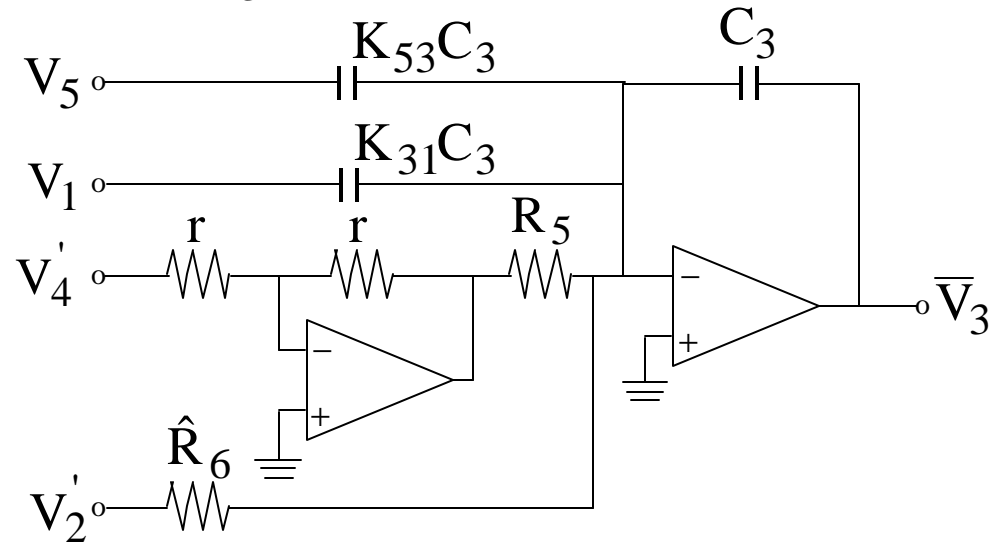
$$V_2' = \frac{R}{SL_{2n}} [V_1 - V_3] \quad (2)$$

$$V_2' = \frac{R}{SL_{2n}} [V_1 + \bar{V}_3] \quad (2')$$

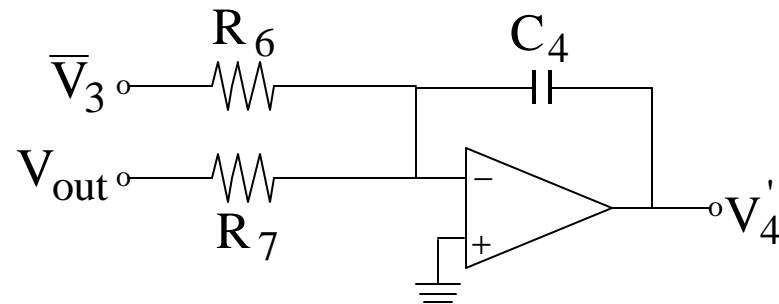


$$V_3 = \frac{1}{\text{SRC}_{234n}} [V_2' - V_4'] + K_{53} V_5 + K_{31} V_1 \quad (3)$$

$$\bar{V}_3 = \frac{1}{\text{SRC}_{234n}} [V_4' - V_2'] - K_{53} V_5 - K_{31} V_1 \quad (3')$$



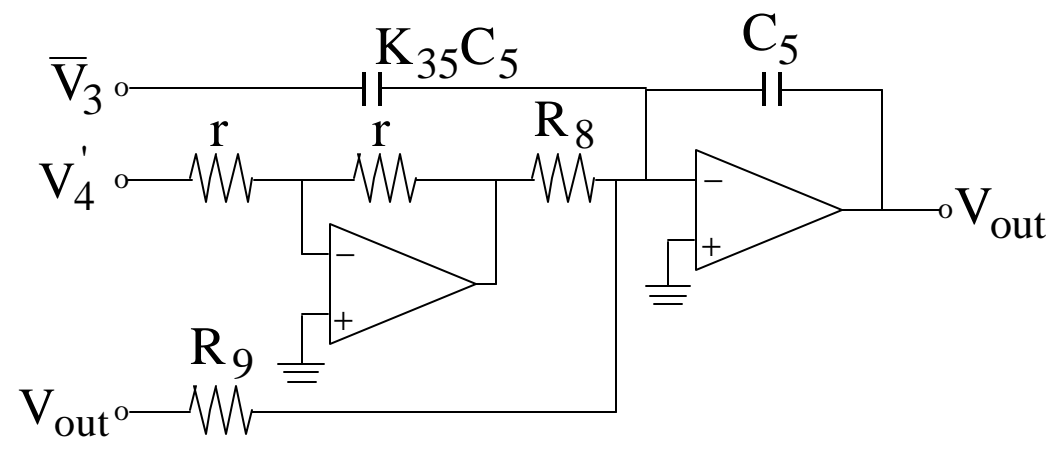
$$V_4' = \frac{R}{\text{SL}_{4n}} [-\bar{V}_3 - V_{\text{out}}] \quad (4)$$

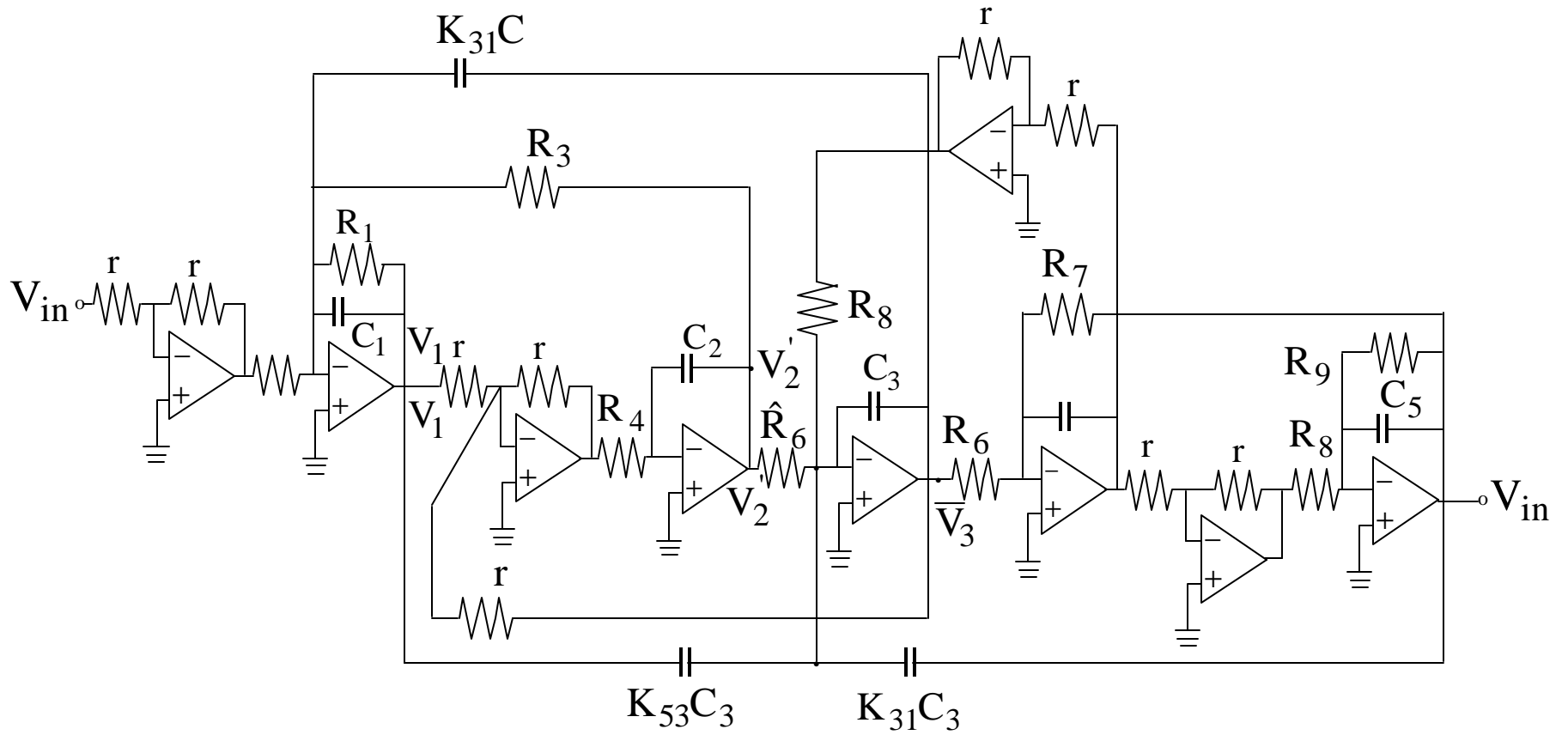


$$R_6 C_4 = L_{4n} / R$$

$$R_7 C_4 = L_{4n} / R$$

$$V_{\text{out}} = V_6 = \frac{1}{SC_{45n}} \left[\frac{V_4'}{R} - \frac{V_{\text{out}}}{R_{6n}} \right] - K_{35} \bar{V}_3 \quad (5)$$

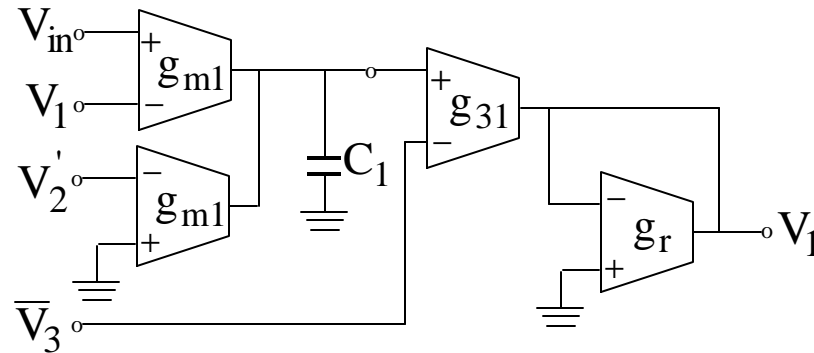




Active RC Fifth-Order Elliptic Filter

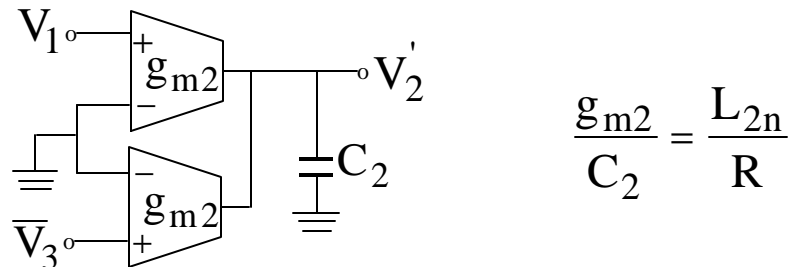
OTA-C Elliptic Implementation

$$V_1 = \frac{1}{SC_{12n}} \left[\frac{V_{in}}{R_{on}} - \frac{V_1}{R_{on}} - \frac{V_2'}{R} \right] + K_{31}V_3 \quad (1)$$

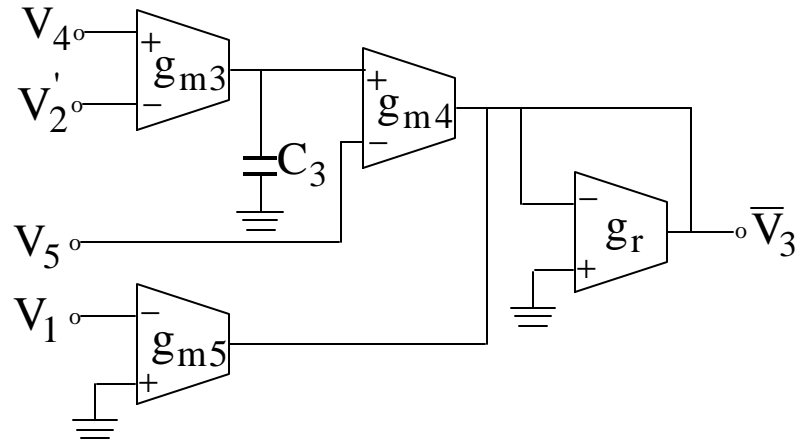


$$\left. \begin{aligned} \frac{g_{31}g_{m1}}{g_r C_1} &= C_{12n}R_{on} \\ \frac{g_{31}}{g_r} &= K_{31} \end{aligned} \right\} \begin{aligned} g_{31} &= K_{31}g_r \\ g_{m1} &= \frac{C_{12n}R_{on}}{K_{31}}C_1 \end{aligned}$$

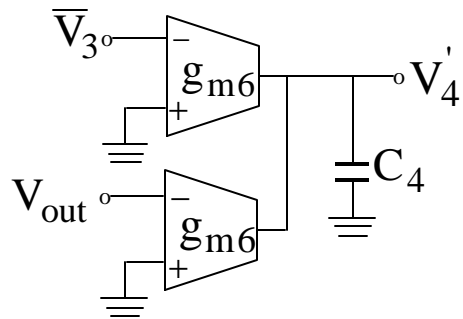
$$V_2' = \frac{R}{SL_{2n}} [V_1 + \bar{V}_3] \quad (2)$$



$$\bar{V}_3 = \frac{1}{sRC_{234n}} [V_4' - V_2'] - K_{53}V_5 - K_{31}V_1 \quad (3)$$

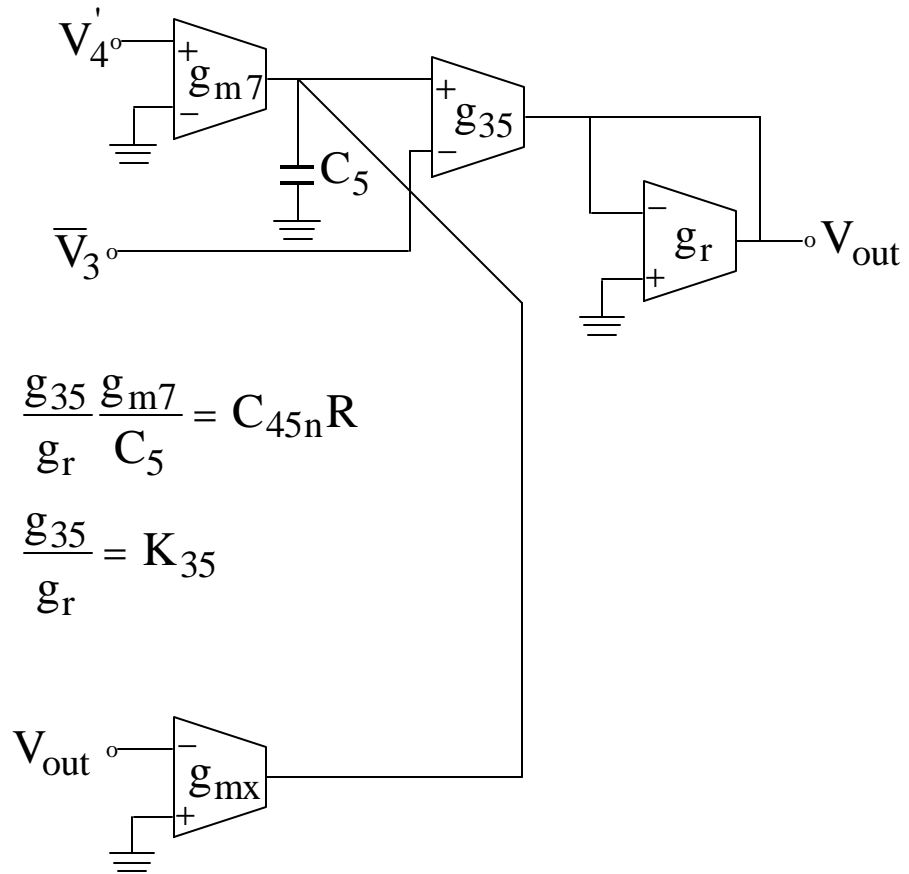


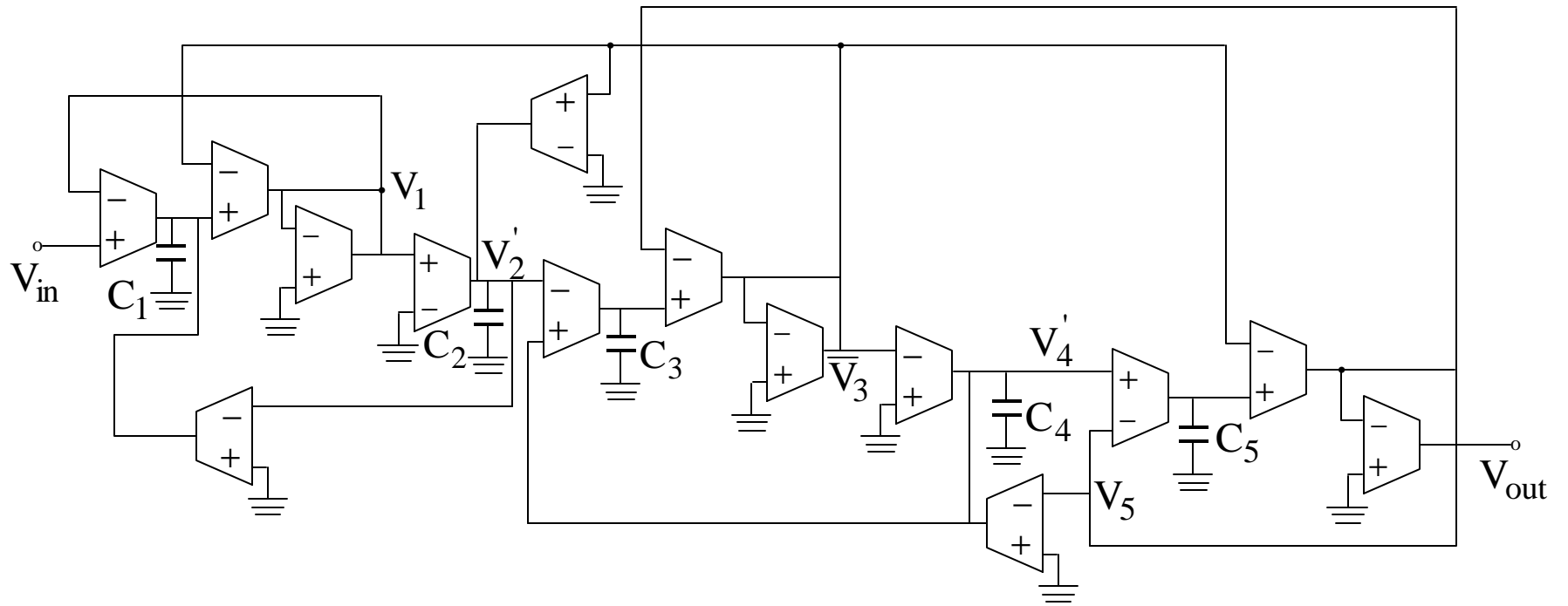
$$V_4' = \frac{R}{sL_{4n}} [-\bar{V}_3 - V_{out}] \quad (4)$$



$$\frac{g_{m6}}{C_4} = L_{4n} / R$$

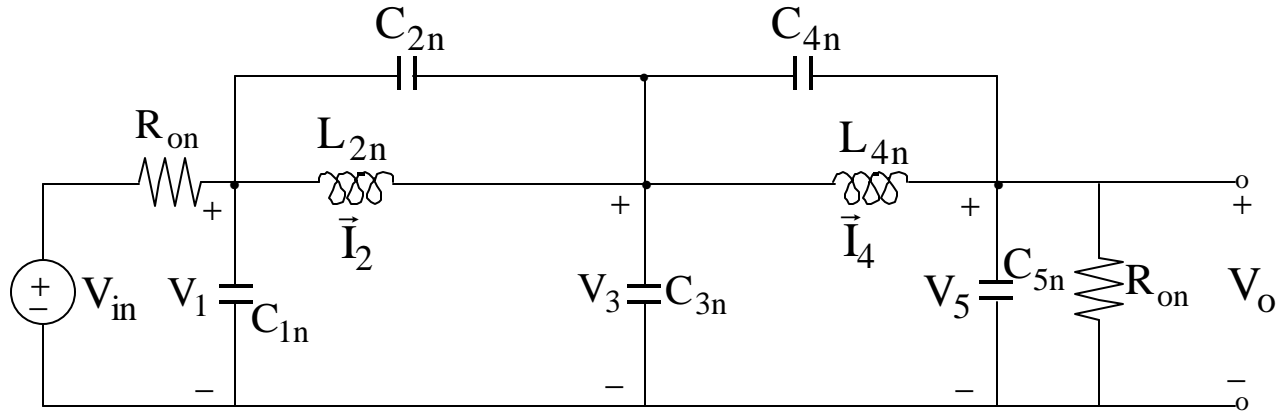
$$V_{\text{out}} = V_5 = \frac{1}{sC_{45n}} \left[\frac{V_4'}{R} - \frac{V_{\text{out}}}{R_{\text{on}}} \right] - K_{35} \bar{V}_3 \quad (5)$$





Fifth-Order OTA-C Elliptic Filter

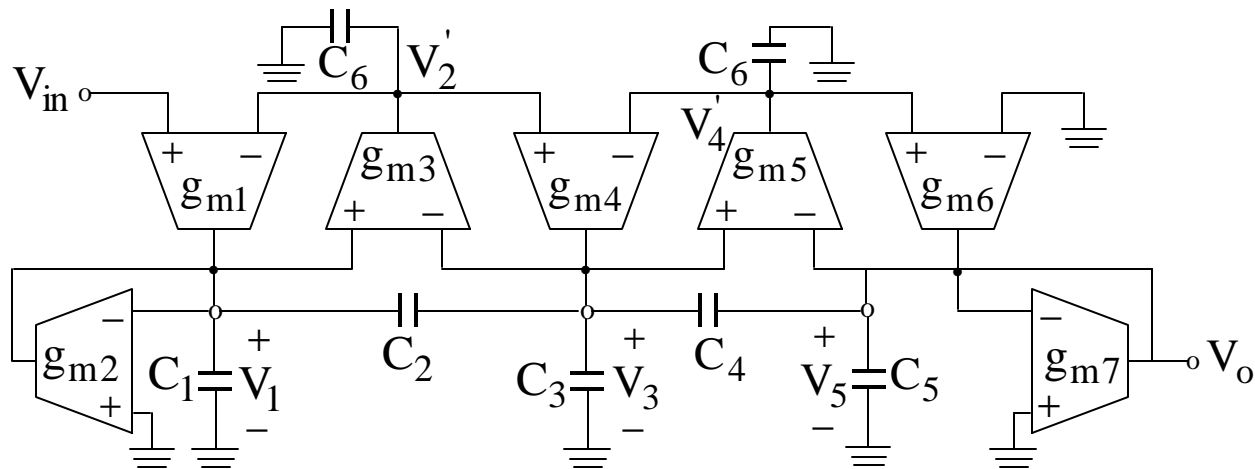
Finite $j\omega$ Zeros: An Alternative Approach



$$sC_{1n} V_1 = \frac{V_{in}}{R_{on}} - I_2 - \frac{V_1}{R_{on}} - sC_{2n} (V_1 - V_3) \quad (1)$$

$$V_2' = I_2 R \quad ; \quad I_2 = \frac{V_2'}{R}$$

$$sC_{1n} V_1 = \frac{V_{in}}{R_{on}} - \frac{V_2'}{R} - \frac{V_1}{R_{on}} - sC_{2n} (V_1 - V_3) \quad (1')$$



$$sL_{2n} \frac{V_2'}{R} = V_1 - V_3 \quad (2)$$

$$sC_{3n} V_3 = \frac{V_2'}{R} - \frac{V_4'}{R} + sC_{2n} (V_1 - V_3) - sC_{4n} (V_3 - V_5) \quad (3)$$

$$s \frac{L_{4n}}{R} V_4' = V_3 - V_5 \quad (4)$$

$$sC_{5n} V_5 = \frac{V_4'}{R} - \frac{V_5}{R_{6n}} + sC_{4n} (V_3 - V_5) \quad (5)$$

Component Calculations

$$\frac{C_1}{g_{m1}} = C_{1n} R_{on}$$

$$R = R_{on}$$

$$g_{m2} = \frac{1}{R_{on}}$$

$$\frac{C_6}{g_{m3}} = \frac{L_{2n}}{R}$$

$$\frac{C_3}{g_{m4}} = RC_{3n}$$

$$\frac{C_7}{g_{m5}} = \frac{L_{4n}}{R}$$

$$g_{m6} = \frac{1}{R}$$

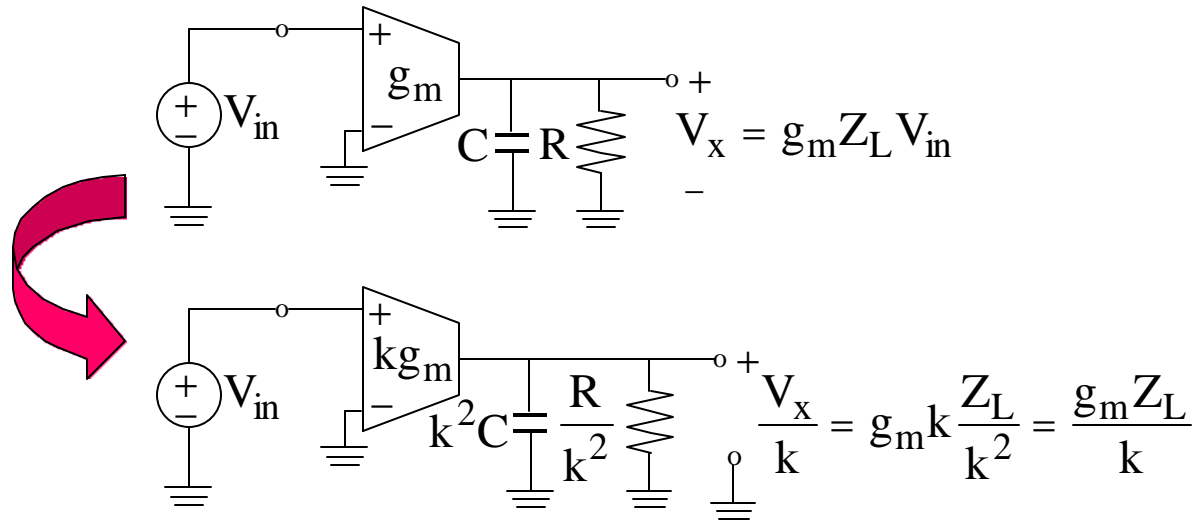
$$g_{m7} = \frac{1}{R_{6n}}$$

Note that if $R_{6n} = R$ then the OTA # 7 can be eliminated and the output of OTA # 6 can be simply connected to its negative terminal.

Voltage Level Scaling Algorithm

$$V_x \rightarrow \frac{1}{k} V_x$$

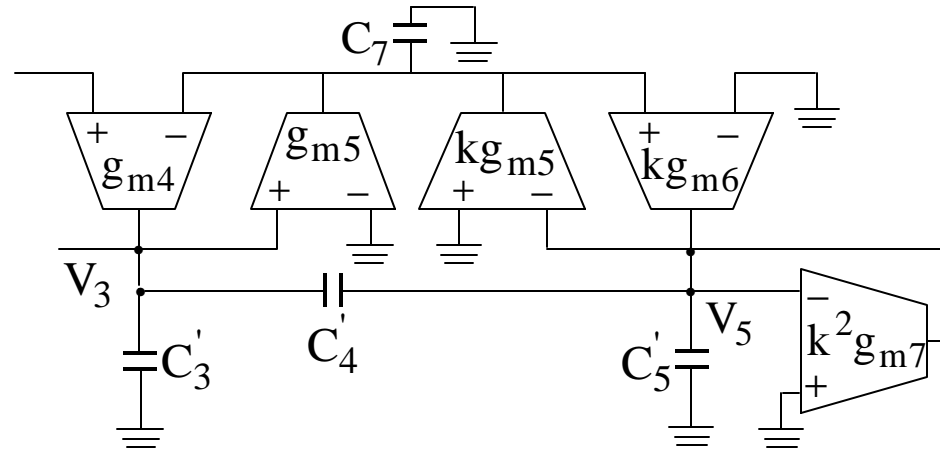
Basic Idea:



- i) Multiply by k^2 the admittance level of the node x in the C subnetwork connected to the node.
- ii) Multiply by k the transconductances of all the OTAs with outputs or inputs connected to the node. In the last case, splitting the OTA in two if the other input is not grounded.

Example

$$V_5 \rightarrow V_5 / k$$



$$C'_3 = C_3 + (1 - k)C_4$$

$$C'_4 = kC_4$$

$$C'_5 = k[kC_5 + (k - 1)C_4]$$

Observe that the total admittance at node 5

$$Y_5 = sC_4 + sC_5 + g_{m7} = s(C_4 + C_5) + g_{m7} \quad \text{Before}$$

$$\bar{Y}_5 = sC'_4 + sC'_5 + k^2g_{m7} = sk^2(C_4 + C_5) + k^2g_{m7} \quad \text{After}$$

At node 3

$$Y_3 = sC_2 + sC_3 + sC_4 \quad \text{Before}$$

$$\bar{Y}_3 = sC_2 + sC'_3 + sC'_4 = sC_2 + sC_3 + sC_4 \quad \text{After}$$