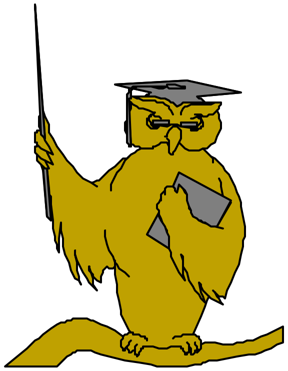
Switched- Capacitor Circuits

TEXAS A&M UNIVERSITY
1876

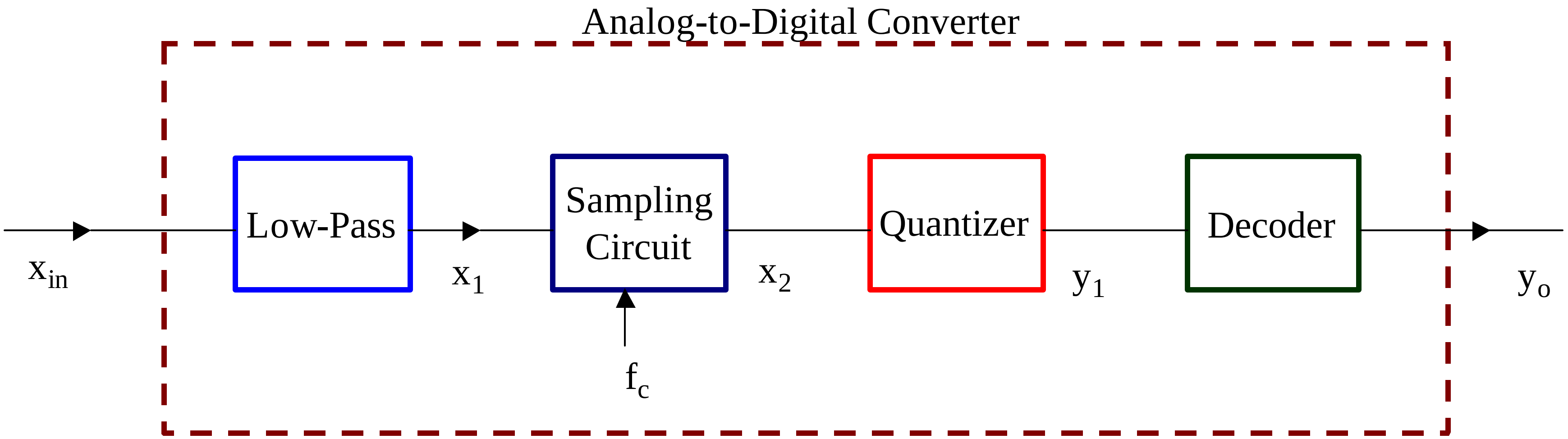
- Background on signals and systems
- Basic Building Blocks
  - Amplifiers
  - Integrators
- Biquadratic Filters
- References



Switched-Capacitor Techniques Advantages

- Reduced silicon area
- Good accuracy. Time constants are implemented with capacitor ratios (~0.1%)
- Don’t require a low-impedance output stage (OTAs could be used)
- Could be implemented using digital circuit process technology
- Very useful in the audio range

Examples of Analog and Digital Signals


Analog-to-Digital Converter

x_{in}

Low-Pass

x_1

Sampling Circuit

f_c

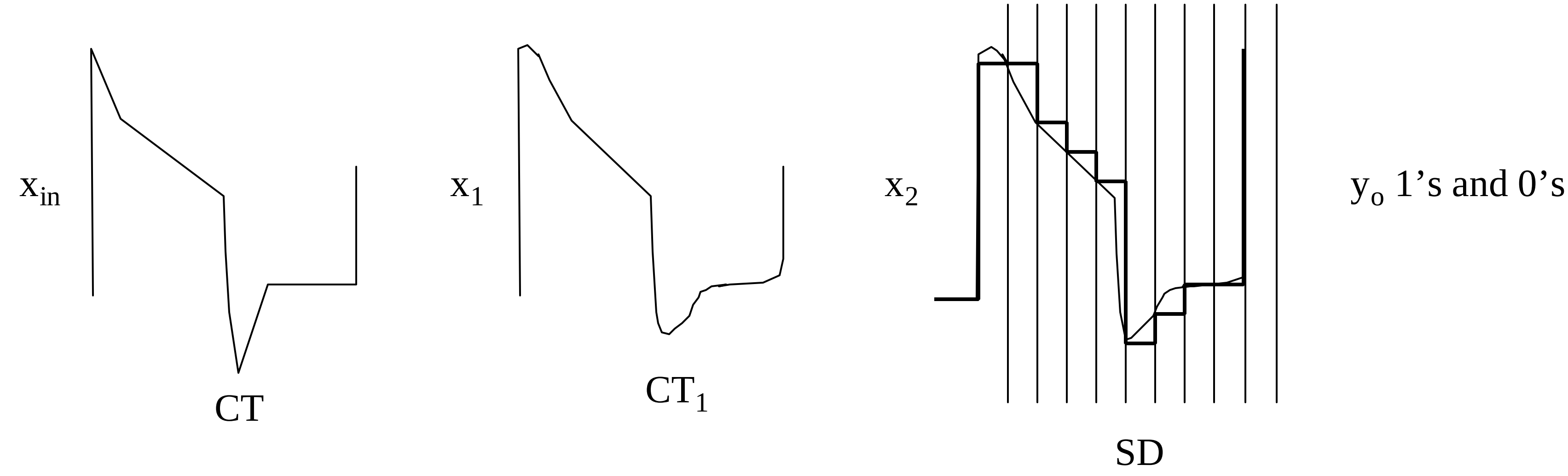
x_2

Quantizer

y_1

Decoder

y_o



Analog Signal and Systems Concepts

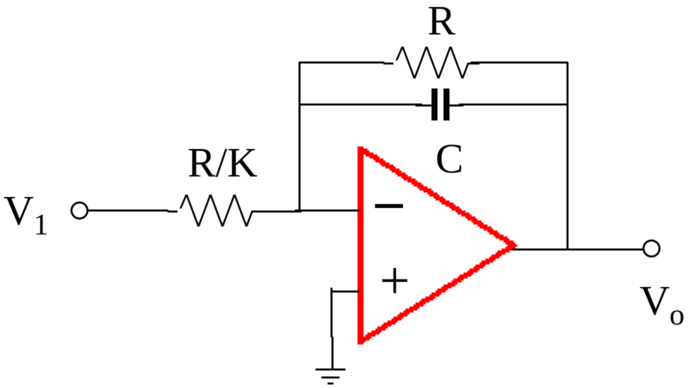
Types of signals	Mathematical Description
• Continuous-Time (CT) --Continuous values in time --Continuous values in magnitude	Laplace
• Sampled Data (SD) --Continuous values in magnitude --Discrete values in time	Z-Transform
• Digital --Discrete values in magnitude --Discrete values in time	Z-Transform

Example of Basic Implementations

- First-order Low Pass

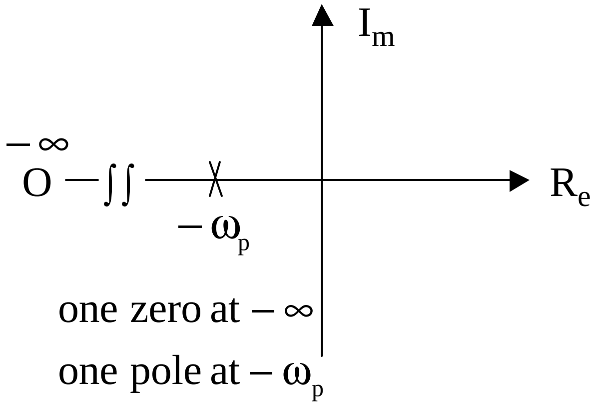
$$H(s)=\frac{K}{1+s/\omega_p} \quad ; \quad \frac{V_o(s)}{V_{in}(s)}=\frac{K\omega_p}{s+\omega_p} \quad ; \quad \frac{dv_o(t)}{dt}+\omega_p v_o(t)=K\omega_p\ v_{in}(t)$$

Continuous-time



$$C=\frac{1}{R\omega_p}$$

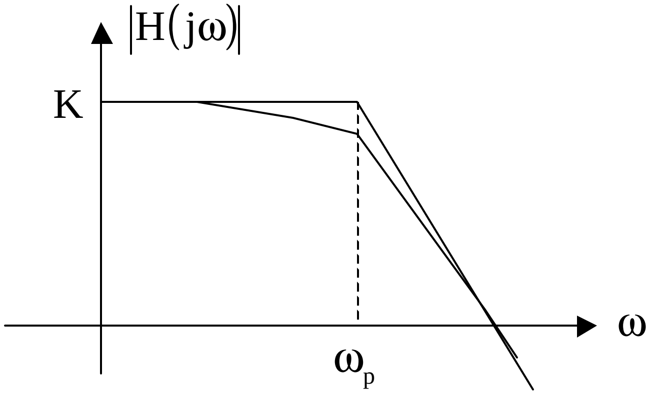
s-plane



one zero at $-\infty$

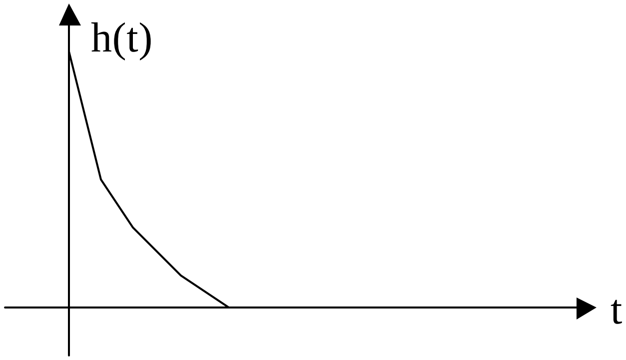
one pole at $-\omega_p$

Stability implies to have poles in the left-half plane (LHP)



Frequency Response

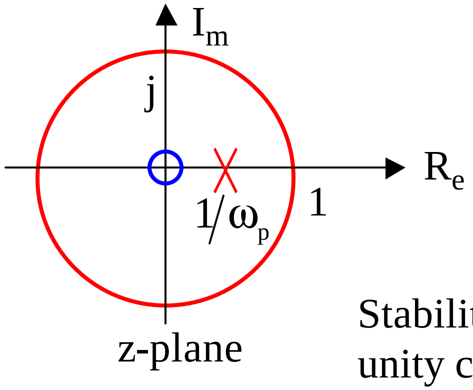
$$H(s)\Big|_{s=j\omega} = H(j\omega) = \frac{-K}{1+j\omega/\omega_p} = \frac{K}{\left[1+(\omega/\omega_p)^2\right]^{1/2}}\angle - \tan\omega/\omega_p$$



Impulse Response

$$h(t) = Kw_p e^{-w_p t}$$

- **First-Order Low-Pass Sampled-Data** (i.e., Switched-Capacitor)



Stability implies poles inside unity circle

$$H(z)=\frac{V_o(z)}{V_{in}(z)}=\frac{K_1}{1-z^{-1}/\omega_{p1}}=\frac{K_1\omega_{p1}}{\omega_{p1}-z^{-1}}\;;$$

$$\left[\omega_{p1}-z^{-1}\right]V_o(z)=K_1\omega_{p1}V_{in}(z)$$

$$\omega_{p1}V_o(z)-z^{-1}V_o(z)=K_1\omega_{p1}V_{in}(z)$$

Taking the inverse z-transform

$$\omega_{p1}v_o(nT)-v_o(n-1)T=K_1\omega_{p1}v_{in}(nT)$$

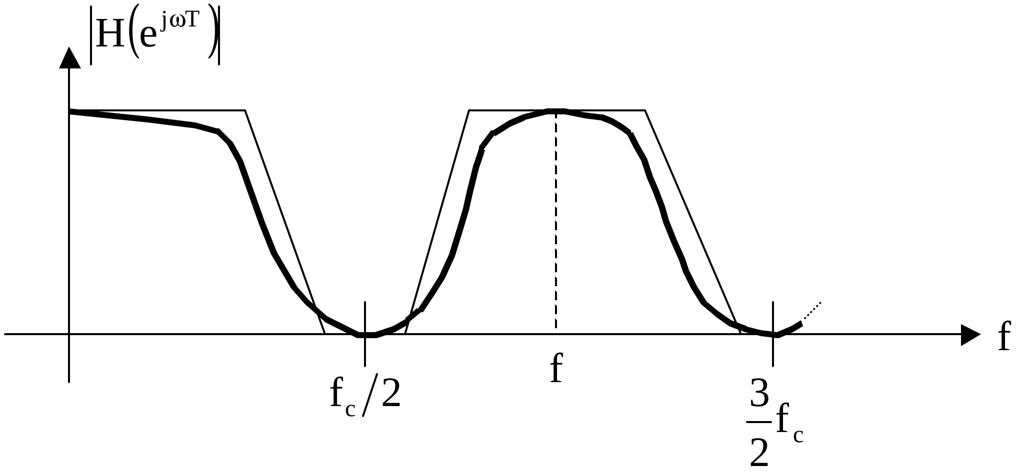
$$v_o(n-1)T-\omega_p v_o(nT)=-K_1\omega_{p1}v_{in}(nT)$$

This difference equation represents the first-order low pass in the Z-domain.

Note that

$$Z^{-1}X_o(z)\Rightarrow x_o(n-1)T$$

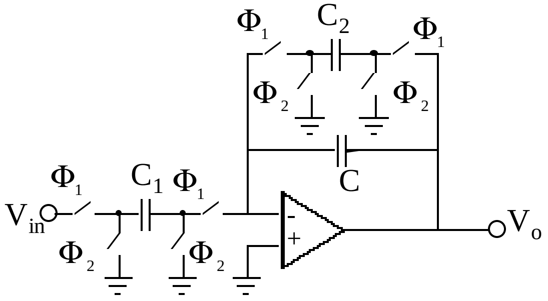
$$Z^{-b}X_o(z)\Rightarrow x_o(n-b)T$$



Frequency Response (A periodic transfer function!)

$$H(z) = \left.\frac{K_1\omega_{p1}Z}{\omega_{p1}Z-1}\right|_{z=e^{j\omega}} = \frac{K_1\omega_{p1}(\cos\omega T + j\sin\omega T)}{(\omega_{p1}\cos\omega T - 1) + j\sin\omega T}$$

$$H(e^{j\omega}) = \frac{K_1\omega_{p1}}{\left\{(\omega_{p1}\cos\omega T - 1)^2 + \sin^2\omega T\right\}^{1/2}}\left[\overbrace{\omega T - \tan^{-1}\frac{\sin\omega T}{\omega_{p1}\cos\omega T - 1}}^{\text{phase}}\right]$$



where

$$K_1\omega_{p1} = \frac{C_1}{C}$$

$$\omega_{p1} = 1 + \frac{C_2}{C}$$

What are the relationships between the s-plane and z-plane?

- There are a number of mappings between the two planes. All these mappings require frequency prewarping to yield quasi identical transfer functions in both planes. That is magnitude and phase of the two transfer functions should be as close as possible.
- The most popular and exact is the bilinear mapping.

$$s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} = \frac{2}{T}\frac{z-1}{z+1} \quad \text{or} \quad z = \frac{1+(T/2)s}{1-(T/2)s}$$

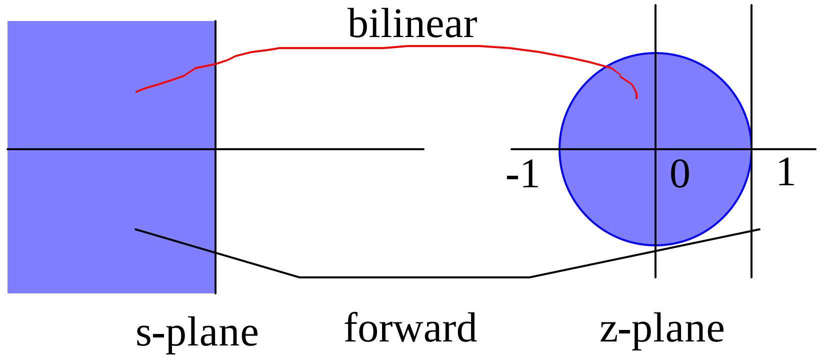
Pre-warping is needed:
W = (2/T)arctan(wT/2)

- The commonly used mapping from s - to z-plane, for high-sampling rate is the **forward mapping** :

$$s = \frac{1}{T}\frac{1-z^{-1}}{z^{-1}} = \frac{1}{T}\frac{z-1}{1}$$

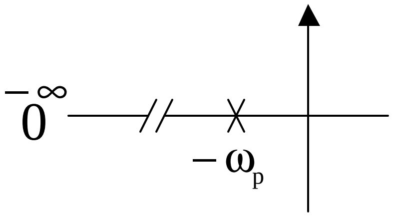
or

$$z = sT + 1$$

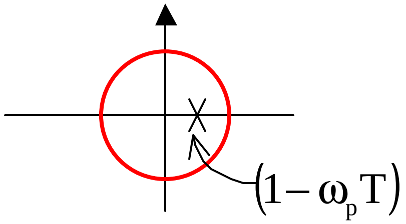


Example. Approximate a first-order low-pass continuous-time to a discrete-time low-pass under high-sampling conditions. $(fs/f >> 1)$

$$H(s) = \frac{K}{1 + s/\omega_p} = \frac{K\omega_p}{s + \omega_p} \Bigg|_{s = \frac{1}{T}(z-1)}$$



$$H(z) = \frac{K\omega_p T}{z - 1 + \omega_p T} = \frac{K\omega_p T}{z - (1 - \omega_p T)}$$



What is the 3dB cut-off frequency in both domains?

$f_{3dB} = \omega_p$ CT

For H(z) is more complex than the computation of f_{3dB}.

$$H\left(e^{j\omega T}\right) = \frac{K\omega_p T}{\cos\omega T + j\sin\omega T - (1 - \omega_p T)}$$

$$\left|H\left(e^{j\omega T}\right)\right|_{\omega = \omega_{3dB}} = \left|H\left(e^{j\omega_{3dB} T}\right)\right| / \sqrt{2}$$

$$\omega_{3dB} = \frac{1}{T}\cos^{-1}\frac{2 - 2\omega_p T - (\omega_p T)^2}{2(1 - \omega_p T)}$$

Numerical example. Low-Pass (First Order)

$$\omega_p = 2\pi\times10^3 \ \mathrm{r/s}$$

$$f_c = f_s = 20\mathrm{KHz} \quad ; \quad T = 1/f_s \quad ; \quad \omega_p T = \frac{2\pi\times10^3}{20\times10^3} = 0.1\pi$$

$$K = 2$$

For the continuous-time

$$f_{3dB} = 1\mathrm{KHz}$$

$$H(j\omega)\Big|_{\omega=\omega_{3dB}} = \frac{2}{\sqrt{2}} = 1.4142$$

For the sample-data

$$f_{3dB} = \frac{f_s}{2\pi}\cos^{-1}\frac{2-2\omega_p T-(\omega_p T)^2}{2(1-\omega_p T)}$$

$$f_{3dB} \cong 1.16\mathrm{KHz}$$

Switched - Capacitor Filters

- Use Z-transform mathematics

- Are described by difference equations

- Time constants are proportional to capacitor ratios

- There are well defined mappings between the s- and z-plane which involve some frequency warping

- Originally the basic goal was to replace resistors by switches and capacitors. Now we focus on how to implement H(z)

- This design approach is one of the most popular in the industry. Basic components are switches, capacitors and Op Amp (usually OTAs)

$$H(z)=\frac{b_2z^{-2}+b_1z^{-1}+b_o}{1-a_1z^{-1}+a_2z^{-2}}\Bigg|_{z=\hat{z}+1}=\frac{\beta_2\hat{z}^{-2}+\beta_1\hat{z}^{-1}+\beta_o}{1+\alpha_1\hat{z}^{-1}+\alpha_2\hat{z}^{-2}}$$

$$\beta_o=b_o,\ \beta_1=2b_o+b_1,\ \beta_2=b_1+b_2+b_o$$

$$\alpha_1=2-a_1,\ \alpha_2=1+a_2-a_1$$

Table Generic Biquadratic Transfer Functiond in the $\hat{z}$- and z-domains, a_1= 2rcosQ and a_2 = r^2, a_1=a and a_2=b.

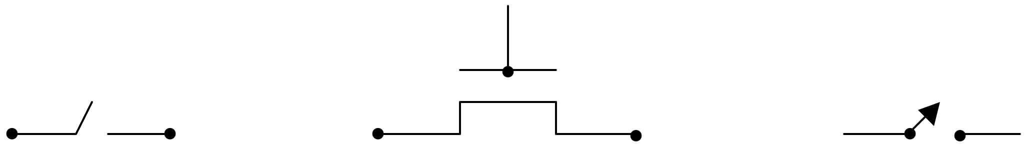
Generic Form	Numerator N(z)	Numerator N($\hat{z}$)
LP 20 (bilinear transform)	$K(1+z^{-1})^2$	$K(1+4\hat{z}^{-1}+4\hat{z}^{-2})$
LP 11	$Kz^{-1}(1+z^{-1})$	$K(2\hat{z}^{-1}+1)\hat{z}^{-1}$
LP 10	$K(1+z^{-1})$	$K(2\hat{z}^{-2}+3\hat{z}^{-1}+1)$
LP 02 (forward transform)	Kz^{-2}	$K\hat{z}^{-2}$
LP 01	Kz^{-1}	$K(1+\hat{z}^{-1})\hat{z}^{-1}$
LP 00 (backward transform)	K	$K(1+\hat{z}^{-1})^2$
BP 10 (bilinear transform)	$K(1-z^{-1})(1+z^{-1})$	$K(1+2\hat{z}^{-1})$
BP 01 (forward)	$Kz^{-1}(1-z^{-1})$	$K\hat{z}^{-1}$
BP 00 (backward)	$K(1-z^{-1})$	$K(1+\hat{z}^{-1})$
HP	$K(1-z^{-1})^2$	K
LPN	$K(1+ez^{-1}+z^{-2})$, $e > a/\sqrt{b}$, $b > 0$	$K[1+\hat{z}^{-1}(2+e)+(2+e)\hat{z}^{-2}]$
HPN	$K(1+ez^{-1}+z^{-2})$, $e < a/\sqrt{b}$, $b > 0$	$K[1+\hat{z}^{-1}(2+e)+(2+e)\hat{z}^{-2}]$
AP	$K(b+az^{-1}+z^{-2})$	

Advantages

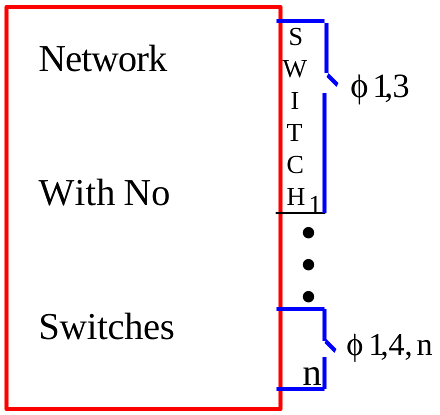
- Have different transfer functions in the z-plane for one transfer function in s-plane.
- Good accuracy. Time constants are implemented with capacitor ratios (~0.1%)
- Don't require a low-impedance output stage (OTA's could be used)
- Could be implemented using digital circuit process technology
- Very useful in the audio range.

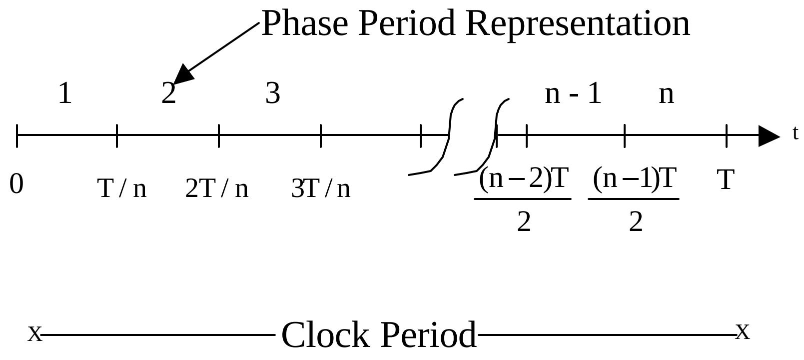
NOTATION

Switches



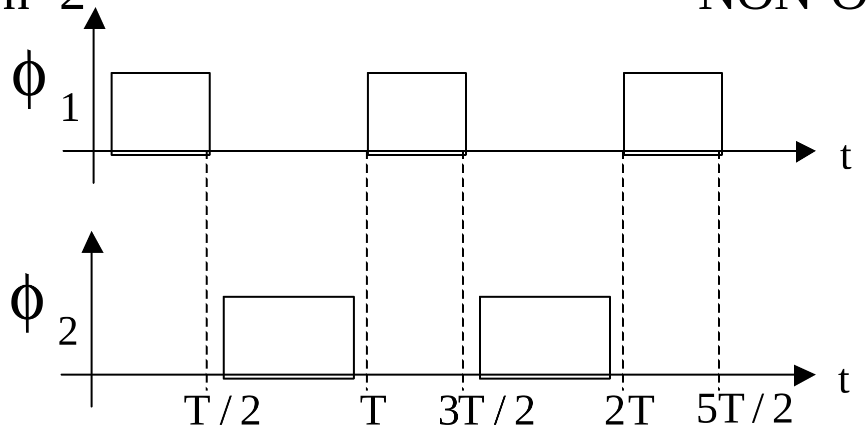
Representation



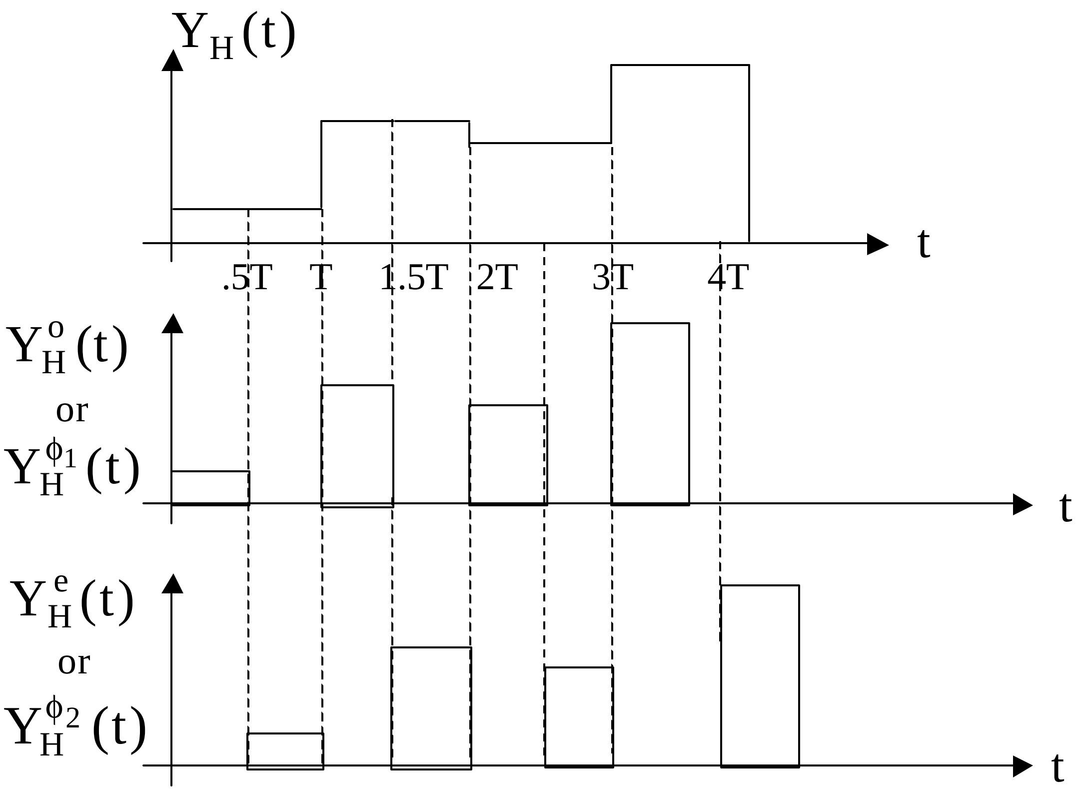


EXAMPLE n=2

NON-OVERLAPPING



PHASE PERIODS OF A CLOCK SEQUENCE

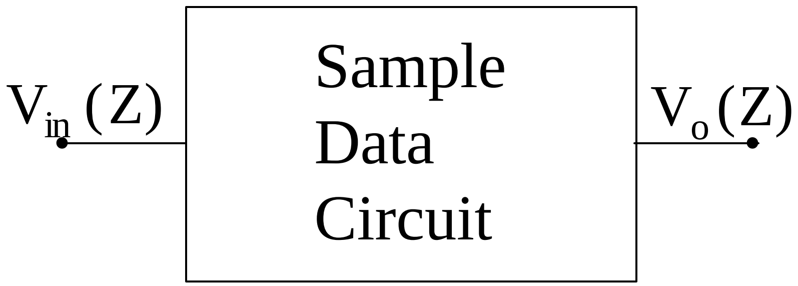


S/H and its respective odd and even components

$$y_H(t) = y_H^o(t) + y_H^e(t)$$

or

$$Y_H(Z) = Y_H^o(Z) + Y_H^e(Z)$$



$V_{in}(Z)$

Sample
Data
Circuit

$V_o(Z)$

$$V_{in}(Z) = V_{in}^{e}(Z) + V_{in}^{o}(Z)$$

$$V_o(Z) = V_o^{e}(Z) + V_o^{o}(Z)$$

$$H(Z) = \frac{V_o(Z)}{V_{in}(Z)}$$

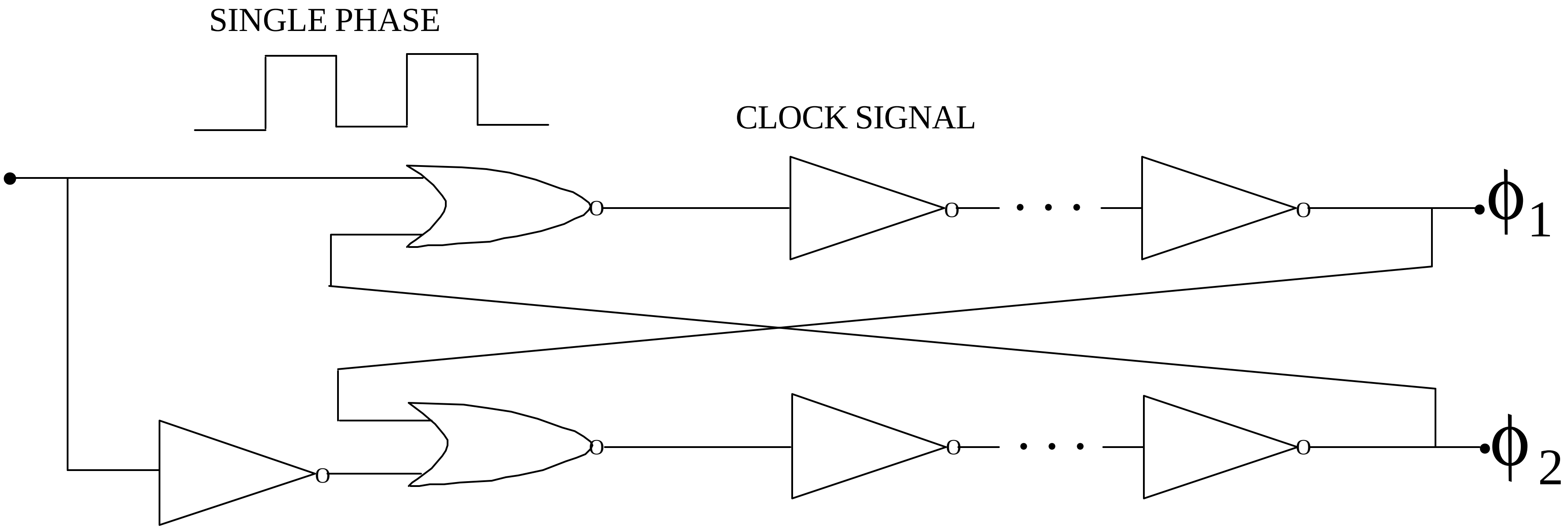
CONVENTIONAL NOTATION FOR TRANSFER FUNCTION IS:

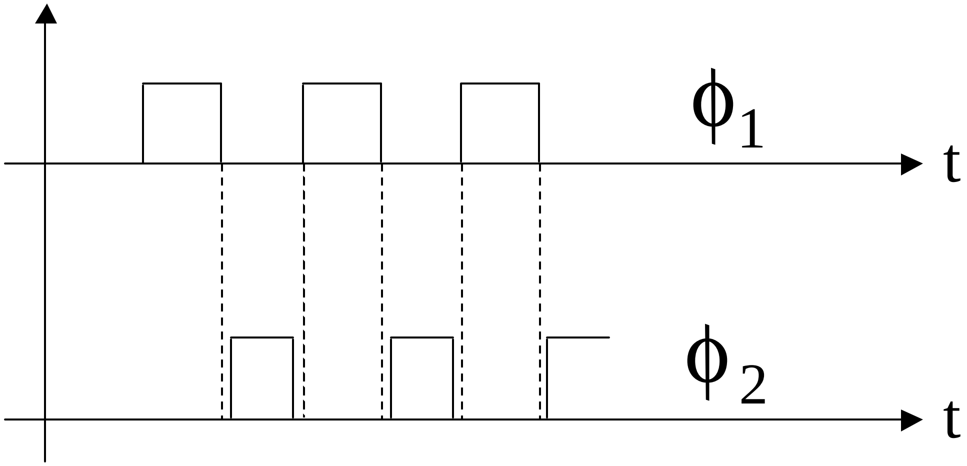
$$H^{ij}(Z) = \frac{V_o^{j}(Z)}{V_{in}^{i}(Z)}$$

i and *j* can be either “e” or “o”. i,e.,

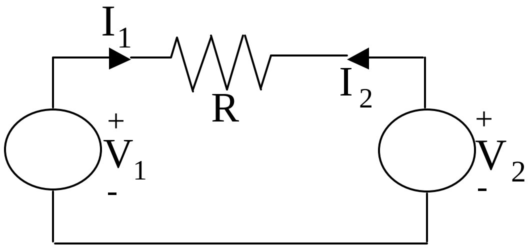
$$H^{eo}(Z) = \frac{V_o^{o}(Z)}{V_{in}^{e}(Z)}$$

TWO-PHASE CLOCK GENERATOR





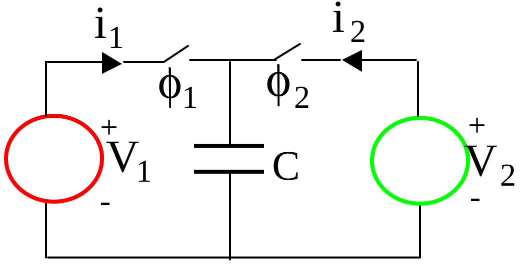
SWITCHED-CAPACITOR EQUIVALENT RESISTOR

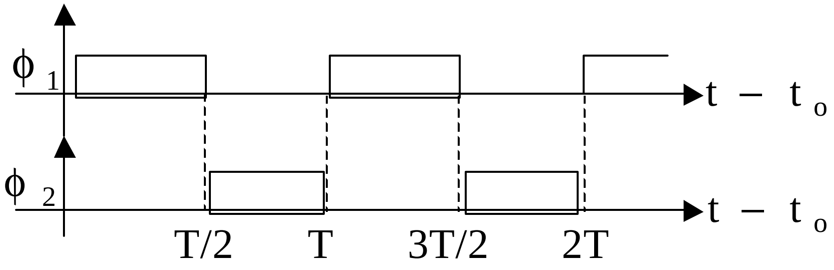


Continuous (Conventional) Resistor

$$R = \frac{V_1 - V_2}{I_1} = \frac{V_2 - V_1}{I_2} \Rightarrow I_2 = \frac{V_2 - V_1}{R}$$

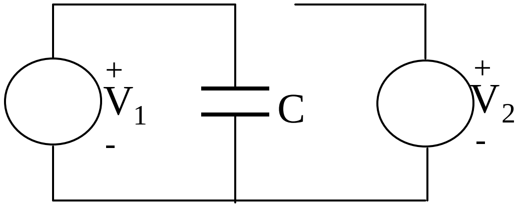
V_1 and V_2 are constant voltage sources





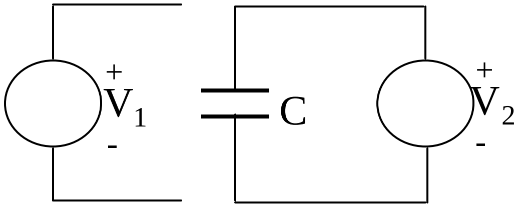
At time $t = t_o$ we apply the clocks.

At ϕ_1



$$Q(t_o + T/2) = CV_1$$

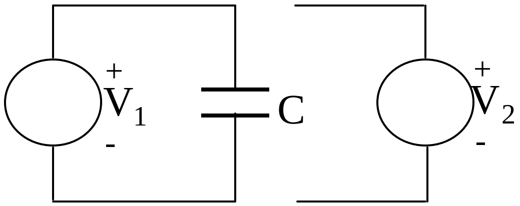
At ϕ_2



$$Q(t_o + T) = CV_2 - CV_1$$

$$Q(t_o + T) = C(V_2 - V_1)$$

For the next period, at ϕ_1



$$Q(t_o + \frac{3T}{2}) = C(V_1 - V_2)$$

$$i = \frac{dQ}{dt}$$

$$Q_1 = \int_{t_o + T}^{t_o + 3T/2} i_1(t)\,dt = \int_{t_o + T/2}^{t_o + 3T/2} i_1(t)\,dt$$

The average current $I_{1\,(aver)}$ becomes

$$I_{1\,(aver)} = \frac{Q\left(t_o + \frac{3T}{2}\right)}{T}$$

$$I_{1\,(aver)} = \frac{1}{T}\int_{t_o+3T/2}^{t_o+3T/2} i_1(t)\,dt$$

$$I_{1\,(aver)} = \frac{C\,(V_1 - V_2}{T} = \frac{\Delta Q}{\Delta t}$$

or

$$\frac{T}{C} = \frac{V_1 - V_2}{I_{1\,(aver)}}$$

Comparing with the continuous time resistor

$$R_{eq} = \frac{T}{C} = \frac{1}{f_C C}$$

EXAMPLE. $R = 250K\Omega, \quad f_C = 128KHz$

$$\frac{A_R}{A_C} \cong 32$$

$$C = \frac{1}{R_{eq} f_C} = \frac{1}{250 \times 128 \times 10^6} = 31.25pF$$

	Continuous R	"SC-R"	
AREA	5,776	178.57	$mils^2$

ACCURACY OF TIME CONSTANTS

- CONTINUOUS TIME:

$$\tau = R_1 C_2$$

$$\frac{d\tau}{\tau} = \frac{dR_1}{R_1} + \frac{dC_2}{C_2} \rightarrow \pm 40\% \rightarrow \pm 65\%$$

where $\frac{d\tau}{\tau}$ is interpreted as the accuracy of τ.

TEMPERATURE DEPENDANT!

- DISCRETE TIME:

$$\tau = \frac{1}{f_C C_1} \cdot C_2 = T\left(\frac{C_2}{C_1}\right)$$

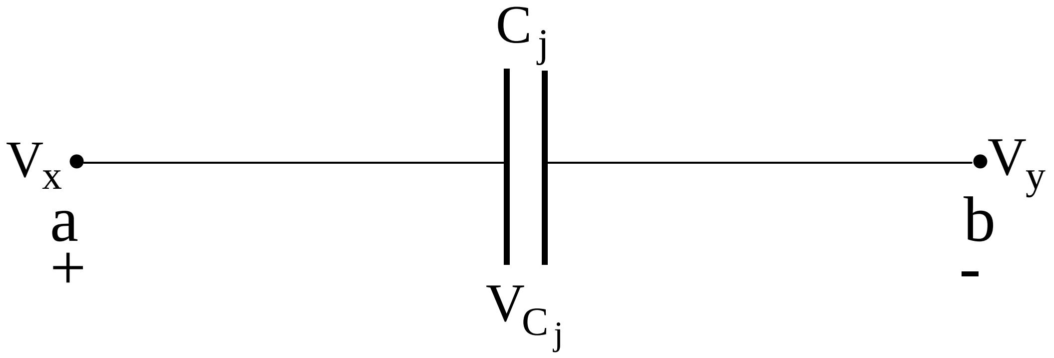
$$\frac{d\tau}{\tau} = \frac{dT}{T} + \frac{dC_2}{C_2} - \frac{dC_1}{C_1}$$

$$\frac{d\tau}{\tau} \cong \frac{dC_2}{C_2} - \frac{dC_1}{C_1} \rightarrow 0.1\%$$ ☺

KC_HL KIRCHOFF “CHARGE” LAW

$$\sum_{i=1}^{n} Q_i = 0$$

$$Q_j = C_j V_{C_j}$$



$t_1 > 0$

$t_2 > t_1$

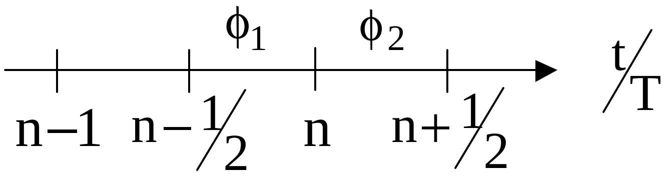
at $t = t_2$

$$V_{C_j} = V_x(t_2) - V_y(t_2) - (V_x(t_1) - V_y(t_1))$$

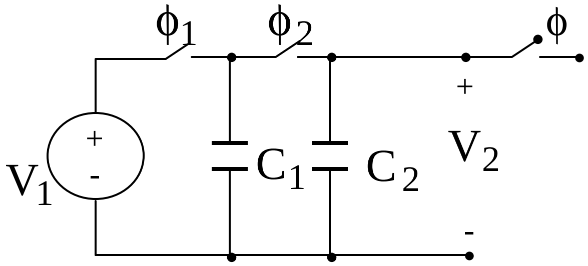
Voltage across the capacitor V_{C_j} = voltage difference (at present time) across the capacitor minus initial condition (at past time).

CHARGE CONSERVATION ANALYSIS METHOD

$$q_L(n) = q_M(n-1) + \hat{q}_C(n)$$



Example



for ϕ_2

$$C_2[V_2^e(n) - V_2^o(n-{}^{1}\!/_{2})] + C_1[V_2^e(n) - V_1^o(n-{}^{1}\!/_{2})] = 0$$

for ϕ_1

$$C_2[V_2^o(n-{}^{1}\!/_{2}) - V_2^e(n-1)] = 0 \Rightarrow V_2^o(n-{}^{1}\!/_{2}) = V_2^e(n-1)$$

THEN

$$C_1 v_2^o(n) + C_2 v_2^o(n) = C_2 v_2^o(n-1) + C_1 v_1^o(n-1)$$

$$(C_1 + C_2) V_2^o(Z) - C_2 Z^{-1} V_2^o(Z) = C_1 V_1^o(Z) Z^{-1}$$

$$H^{oo}(Z) = \frac{\frac{C_1}{C_1} Z^{-1}}{\frac{C_1 + C_2}{C_1} - \frac{C_2}{C_1} Z^{-1}} = \frac{Z^{-1}}{1 + \alpha - \alpha Z^{-1}}$$

$$H^{oo}(Z) = \frac{\frac{C_1}{C_1} Z^{-1}}{\frac{C_1 + C_2}{C_1} - \frac{C_2}{C_1} Z^{-1}} = \frac{Z^{-1}}{1 + \alpha - \alpha Z^{-1}}$$

$$H^{oo}(Z) = \frac{V_o^o(Z)}{V_{in}^o(Z)} = \frac{V_o^e(Z) Z^{-1/2}}{V_{in}^o(Z)}$$

$$V_2^e(n) = V_2^e(n + {}^1\!/_2)$$

$$C_2[V_2^o(n + {}^1\!/_2) - V_2^o(n - {}^1\!/_2)] + C_1[V_2^o(n + {}^1\!/_2) - V_1^o(n - {}^1\!/_2)]$$

$$C_2[V_2^o(Z)][Z^{1/2} - Z^{-1/2}] + C_1 V_2^o(Z) Z^{1/2} = C_1 V_1^o(Z) Z^{-1/2}$$

$$C_2(1 - Z^{-1}) V_2^o(Z) + C_1 V_2^o(Z) = C_1 V_1^o(Z) Z^{-1/2}$$

$$\frac{V_2^o(Z)}{V_1^o(Z)} = \frac{C_1 Z^{-1}}{C_2(1 - Z^{-1}) + C_1} = \frac{C_1 Z^{-1}}{C_2 + C_1 - C_2 Z^{-1}}$$

$$\frac{V_2^o(Z)}{V_1^o(Z)} = \frac{C_1}{(C_2 + C_1)Z - C_2}\Bigg|_{Z \cong 1 + ST} = \frac{C_1}{(C_2 + C_1) + (C_2 + C_1)ST - C_2}$$

For high-sampling rate $\omega T << 1$

$$\frac{V_2^o(Z)}{V_1^o(Z)}\Bigg|_{Z = 1 + sT} = \frac{C_1}{C_1 + (C_2 + C_1)sT} = \frac{1}{1 + \left(\frac{C_2 + C_1}{C_1}\right)sT} = \frac{1}{1 + s \Big/ \frac{C_1}{(C_2 + C_1)T}}$$

$$f_{3d\beta} \cong \frac{1}{2\pi} \cdot \frac{C_1}{(C_2 + C_1)T} = \frac{1}{2\pi} \cdot \frac{f_c}{1 + \frac{C_2}{C_1}}$$

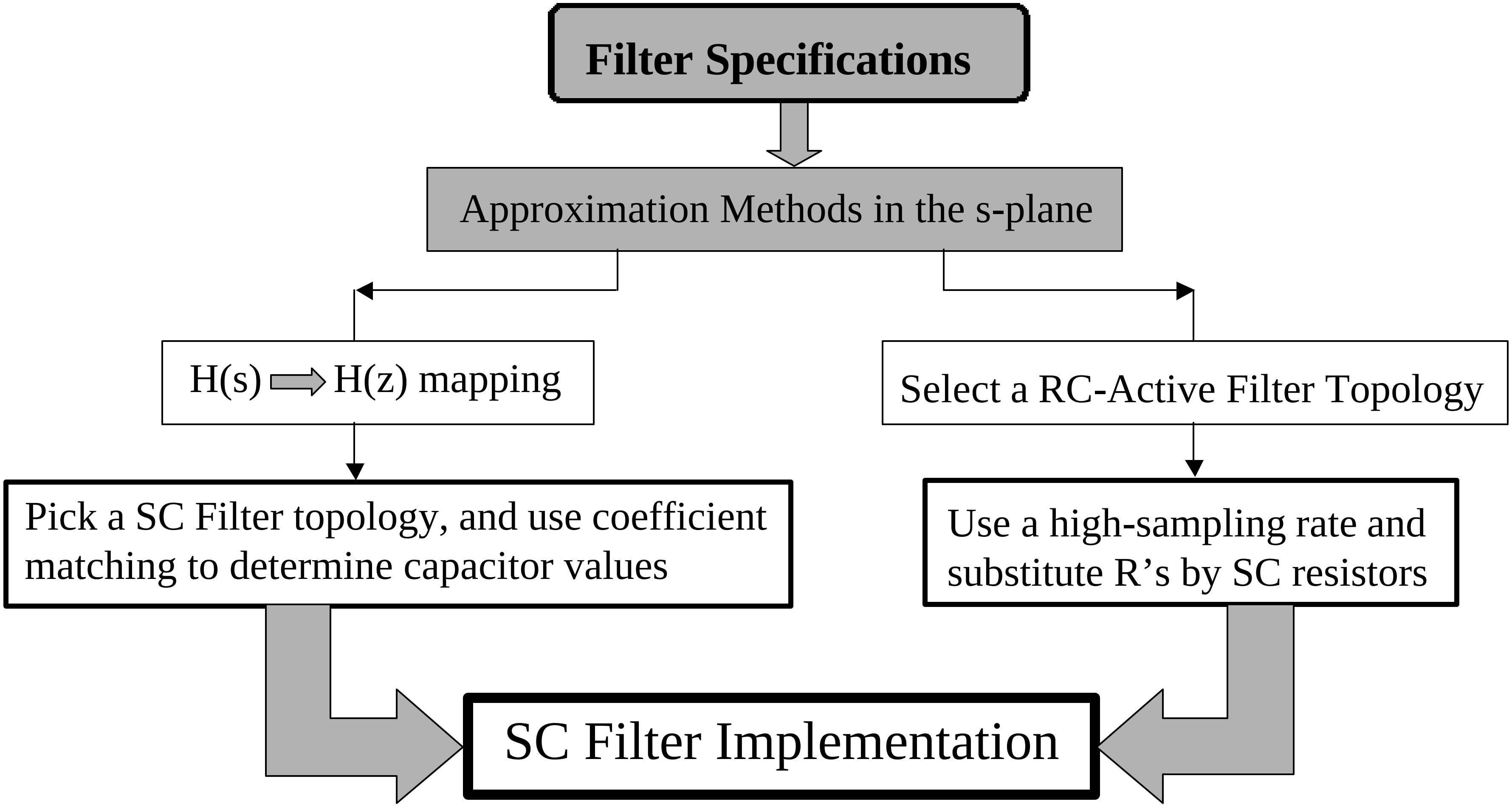
Aside:

$$f_{3d\beta} \cong \frac{1}{2\pi} \cdot \frac{1}{R_1 C_2} \cong \frac{1}{2\pi} \cdot \frac{1}{\frac{TC_2}{C_1}} = \frac{1}{2\pi} \cdot \frac{f_c}{\frac{C_2}{C_1}}$$

Switched-Capacitor (SC) Filters

How to design SC Filters ?

- Two basic approaches



Filter Specifications

Approximation Methods in the s-plane

H(s) ⇨ H(z) mapping

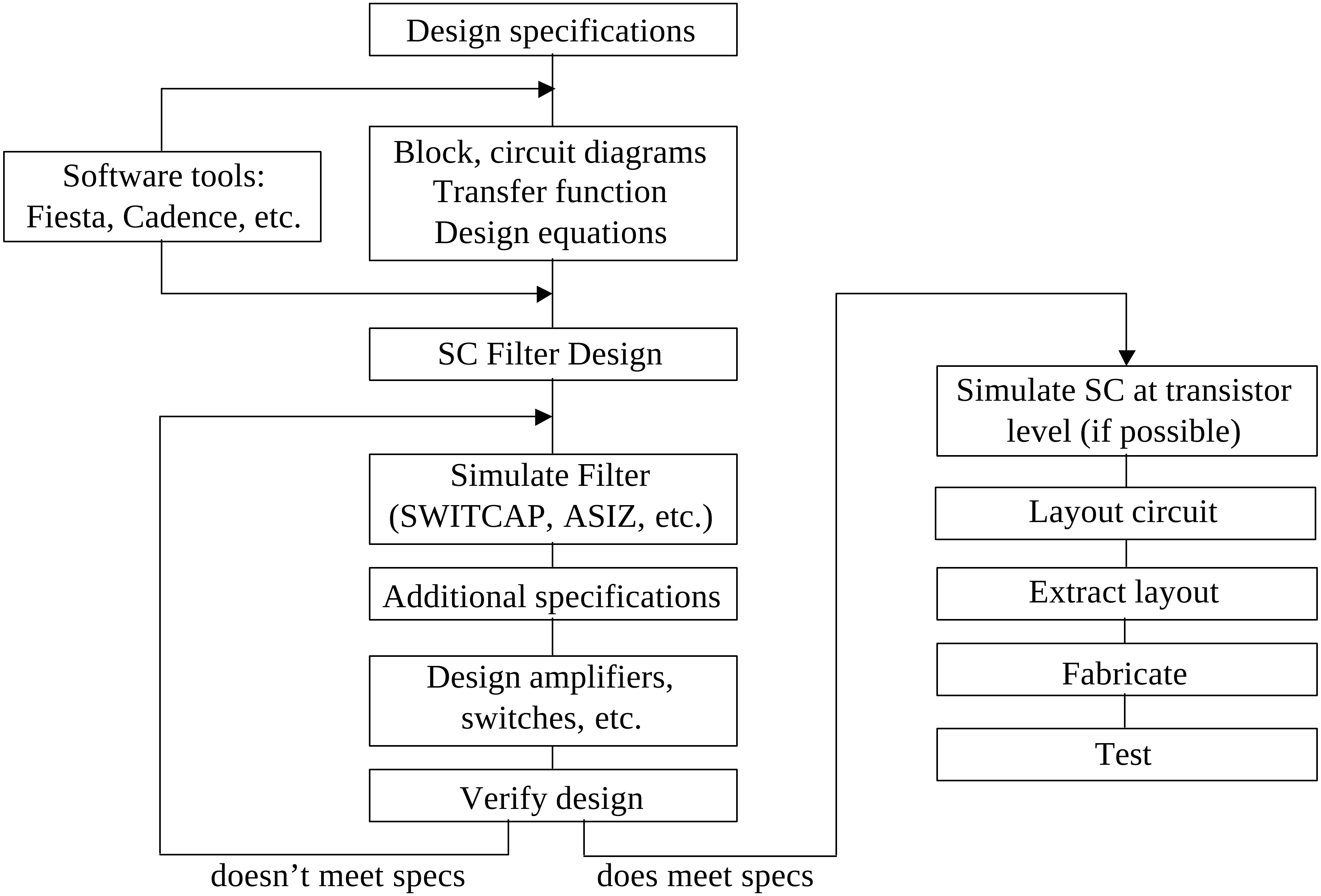
Select a RC-Active Filter Topology

Pick a SC Filter topology, and use coefficient matching to determine capacitor values

Use a high-sampling rate and substitute R's by SC resistors

SC Filter Implementation

Systematic SC Filter Design



Design specifications

Block, circuit diagrams
Transfer function
Design equations

Software tools:
Fiesta, Cadence, etc.

SC Filter Design

Simulate Filter
(SWITCAP, ASIZ, etc.)

Additional specifications

Design amplifiers,
switches, etc.

Verify design

doesn’t meet specs

does meet specs

Simulate SC at transistor
level (if possible)

Layout circuit

Extract layout

Fabricate

Test

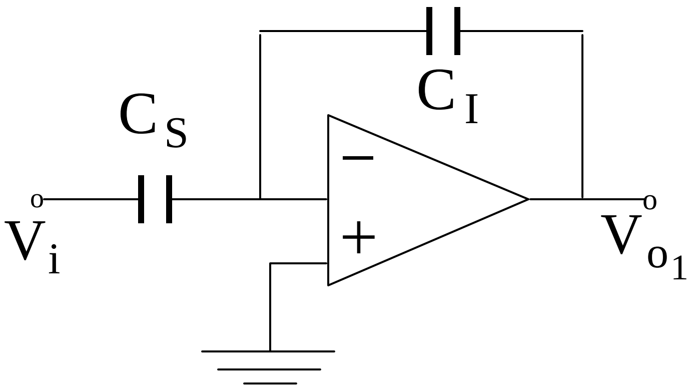
What are the advantages and disadvantages of the two filter design procedures ?

- Mapping Techniques
  - \+ Systematic and well documented (see FIESTA-2)
  - \+ It can use any sampling rate, including the (minimum) Nyquist rate.
  - \- Difficult to implement by hand calculation

- Transforming R to SC resistors
  - \+ It is, conceptually, easier to follow for analog designer
  - \+ Its design is straightforward
  - \- Yields not an optimal design for area, imposed high sampling rate involves larger capacitor ratios.

BASIC BUILDING BLOCKS

A. GAIN AMPLIFIERS



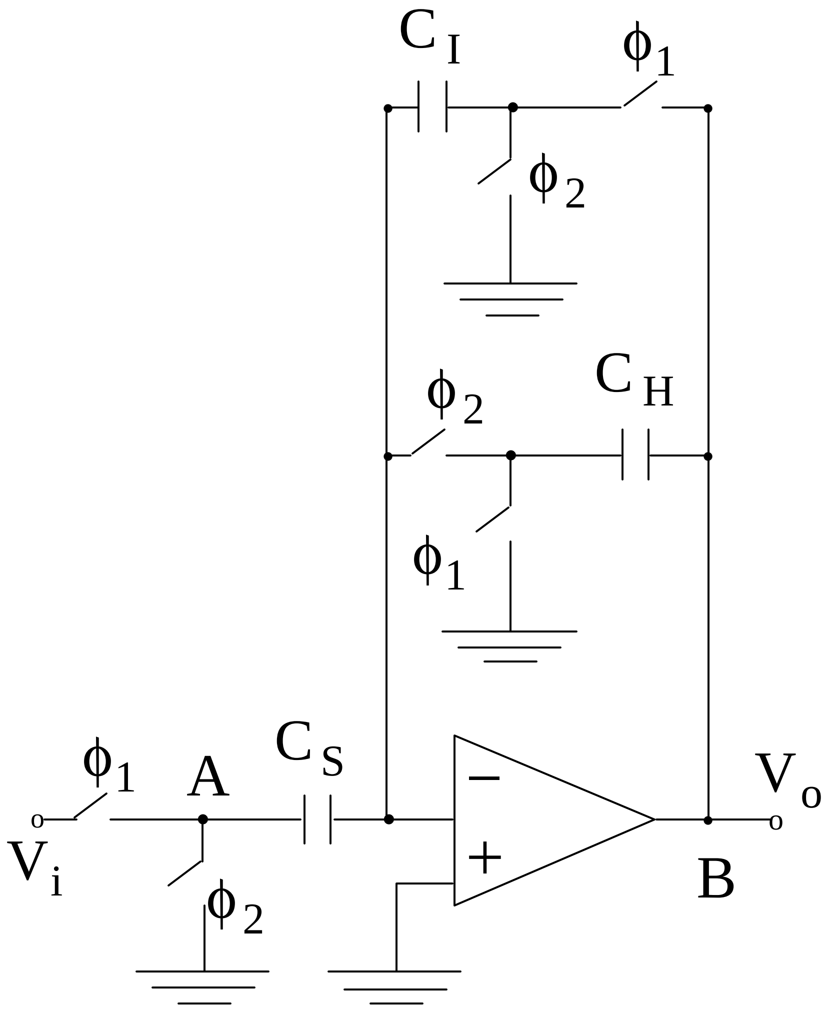
Basic Configuration:

- Compact, versatile & time continuous
- Lacks dc feedback
  - Op amp is not stabilized
  - Leakage current saturates the circuit

$$\text{Gain} = -\frac{C_S}{C_I}$$

Improved Gain Amplifier

- Low sensitivity to the op amp offset voltage and open loop gain (due to charge cancellation)
- Additional capacitor (not shown in Fig.) between nodes A and B eliminates the “spikes” during the non-overlapping clock phases

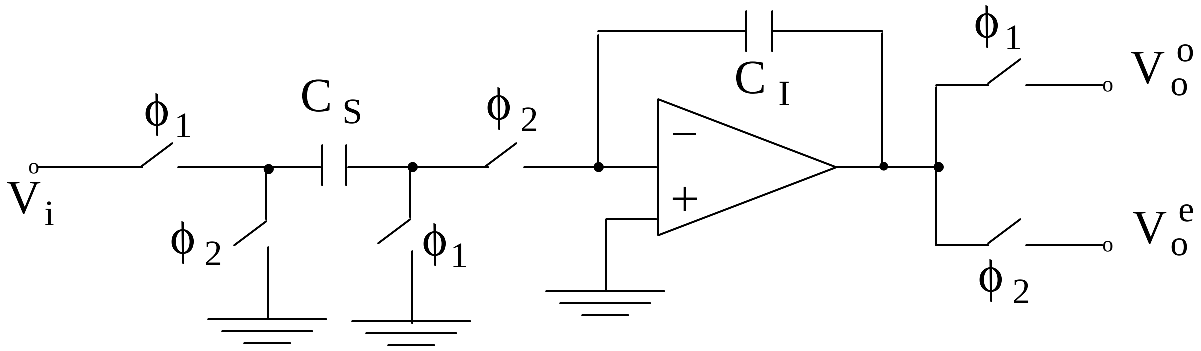


$$\text{gain} = -\frac{C_S}{C_I}$$

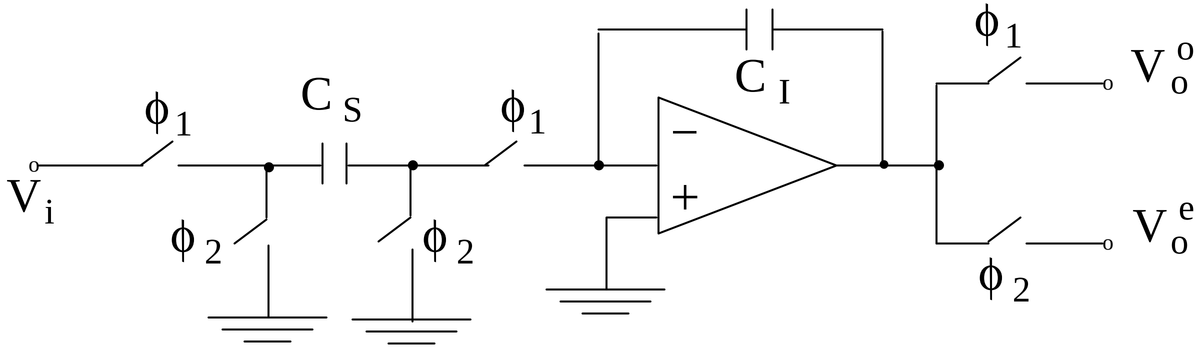
B. Integrators

Conventional stray-insensitive integrators:

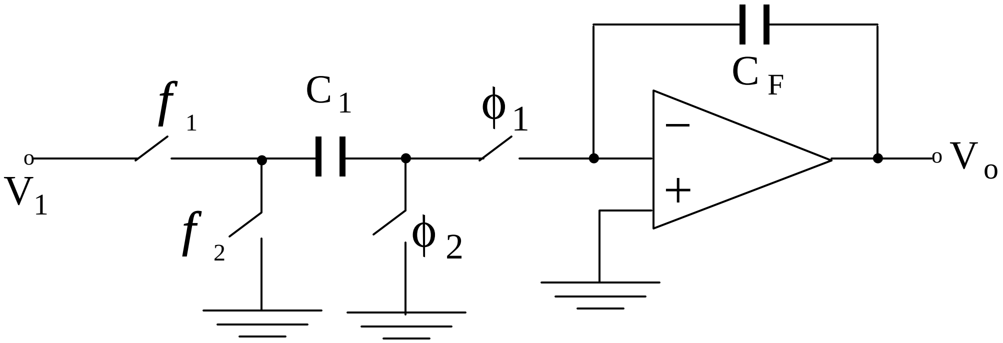
- Non-inverting:

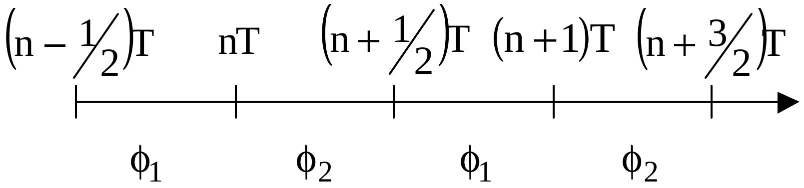


- Inverting:

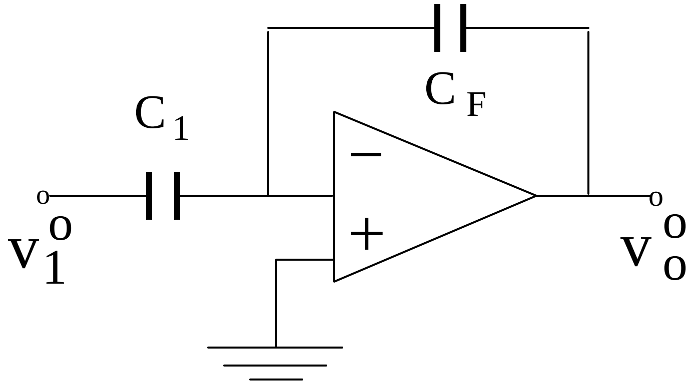


Example of SC Analysis: An Inverting Integrator





During f_1 (odd clock phase) at $(n-1/2) \le t/T<n$

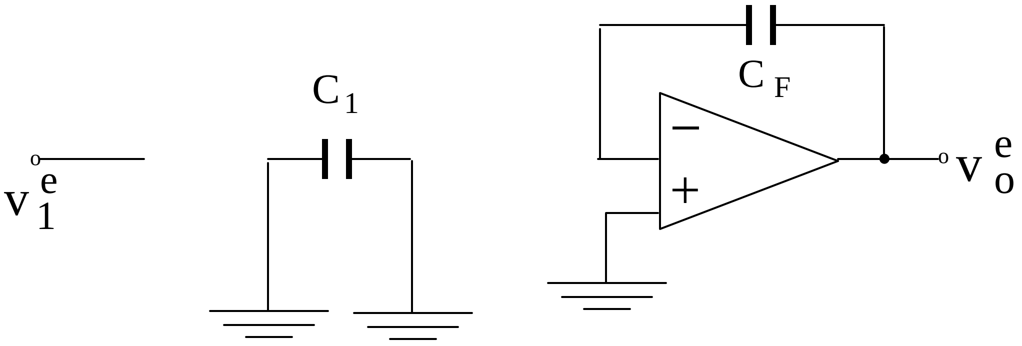


$$v_{C_1} = v_1^o\left(n - \tfrac{1}{2}\right)T - O$$

$$v_{C_F} = v_o^o\left(n - \tfrac{1}{2}\right)T - v_o^e(n-1)T$$

$$C_1 v_{C_1} + C_F v_{C_F} = C_1 v_1^o\left(n-\frac{1}{2}\right)T + C_F v_o^o\left(n-\frac{1}{2}\right)T - C_F v_o^e(n-1)T = 0 \qquad (1)$$

During f_2 (even clock phase) at $n \leq t/T < n + 1/2$



$$v_{C_F} = v_o^e(nT) - v_o^o\left(n - \frac{1}{2}\right)T = 0$$

or

$$v_o^e(nT) = v_o^o\left(n - \frac{1}{2}\right)T \qquad (2)$$

Applying z- transform to both (1) and (2) one obtain

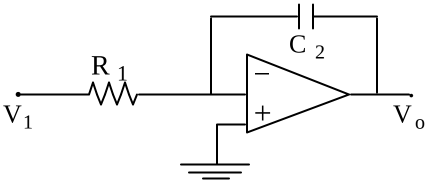
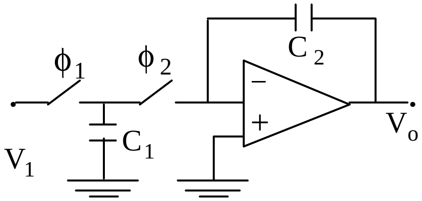
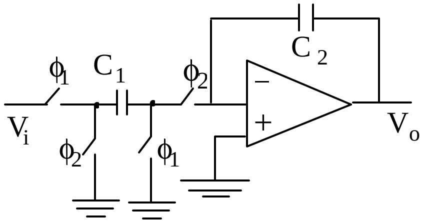
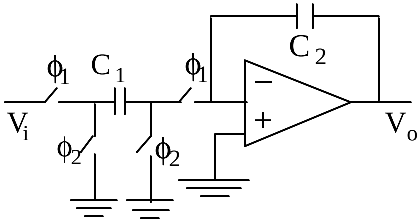
$$C_1 z^{-1/2} V_1^o(z) + C_F z^{-1/2} V_o^o(z) - C_F V_o^e(z) z^{-1} = 0 \qquad (3)$$

$$V_o^e(z) = z^{-1/2} V_o^e(z) \qquad (4)$$

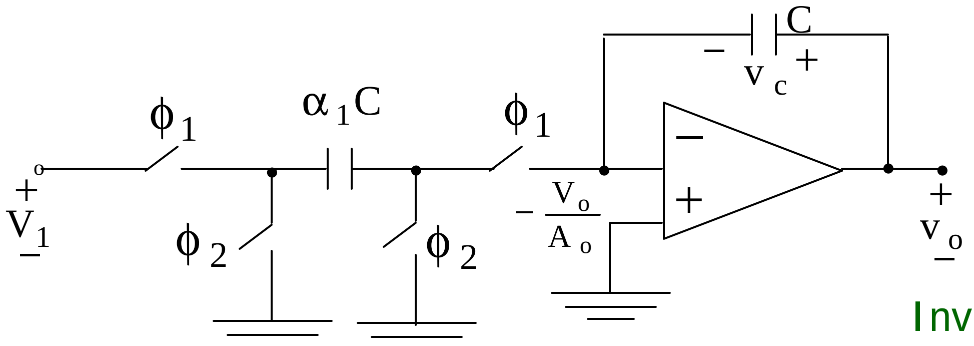
Solving (3) and (4)

$$C_1 V_1^o(z) + C_F V_o^o(z) - C_F z^{-1} V_o^e(z) = 0$$

$$H^{oo}(z) = \frac{V_o^o(z)}{V_1^o(z)} = \frac{-C_1/C_F}{1 - z^{-1}} \quad ; \quad H^{oe}(z) = \frac{V_o^e(z)}{V_1^o(z)} = \frac{-C_1/C_F \, z^{-1/2}}{1 - z^{-1}}$$

Type of Integrator	Magnitude, $\lvert H(e^{j\omega T})\rvert$	Phase, $A_{RG}H(e^{j\omega T})$	Mapping (Equivalent)	Transfer Function
	$\frac{\omega_o}{\omega}$	$\frac{\pi}{2}$	In the s-Plane	i.e. $H(S)=\frac{1}{SR_1C_2}=-\frac{\omega_o}{S}$
 Inverting Stray-sensitive (Forward)	For V_o at ϕ_2 $\frac{\omega_o}{\omega}(\frac{\omega T/2}{\sin(\omega T/2)})$	$\frac{\pi}{2}$	LDI	$H(Z)=-\frac{C_1}{C_2}(\frac{Z^{-1/2}}{1-Z^{-1}})$
	For V_o at ϕ_1 $\frac{\omega_o}{\omega}(\frac{\omega T/2}{\sin(\omega T/2)})$	$\frac{\pi}{2}-\frac{\omega T}{2}$	Forward	$H(Z)=-\frac{C_1}{C_2}(\frac{Z^{-1}}{1-Z^{-1}})$
 Non-Inverting	For V_o at ϕ_2 $\frac{\omega_o}{\omega}(\frac{\omega T/2}{\sin(\omega T/2)})$	$-\frac{\pi}{2}$	LDI	$H(Z)=\frac{C_1}{C_2}(\frac{Z^{-1/2}}{1-Z^{-1}})$
	For V_o at ϕ_1 $\frac{\omega_o}{\omega}(\frac{\omega T/2}{\sin(\omega T/2)})$	$-\frac{\pi}{2}-\frac{\omega T}{2}$	Forward	$H(Z)=\frac{C_1}{C_2}(\frac{Z^{-1}}{1-Z^{-1}})$
 Inverting (Backward)	For V_o at ϕ_1 $\frac{\omega_o}{\omega}(\frac{\omega T/2}{\sin(\omega T/2)})$	$\frac{\pi}{2}+\frac{\omega T}{2}$	Backward	$H(Z)=-\frac{C_1}{C_2}(\frac{1}{1-Z^{-1}})$
	For V_o at ϕ_2 $\frac{\omega_o}{\omega}(\frac{\omega T/2}{\sin(\omega T/2)})$	$\frac{\pi}{2}$	LDI	$H(Z)=-\frac{C_1}{C_2}(\frac{Z^{-1/2}}{1-Z^{-1}})$

DC OPEN LOOP GAIN A_o EFFECTS



Inverting Integrator

f_1 Odd Phase f_2 Even Phase i) During f_1 $\left(n-\frac{1}{2}\right) < \frac{t}{T} \le n:$

$$v_C^o(n) = v_o^o(n) + \frac{v_o^o(n)}{A_o} \qquad (1)$$

Furthermore $\sum_{i=1}^{2} Q_i = 0$

$$\alpha_1 C v^o_{\alpha_1 C} + C v_C^o = 0$$

$$\alpha_1\left[v_1^o(n) + \frac{v_o^o(n)}{A_o}\right] + v_C^o(n) - v_C^e\left(n-\frac{1}{2}\right) = 0 \qquad (2)$$

ii) During f_2 $(n-1) < \frac{t}{T} \le \left(n-\frac{1}{2}\right)$

$$v_C^e\left(n-\frac{1}{2}\right) = v_o^e\left(n-\frac{1}{2}\right) + \frac{v_o^e\left(n-\frac{1}{2}\right)}{A_o} \qquad (3)$$

Eqs (3) and (1) into (2) yields

$$\alpha_1\left[v_1^o(n)+\frac{v_o^o(n)}{A_o}\right]+v_o^o(n)\left(1+\frac{1}{A_o}\right)-v_o^e\left(n-\frac{1}{2}\right)\left(1+\frac{1}{A_o}\right) \qquad (2')$$

Note that

$$v_o^e\left(n-\frac{1}{2}\right)=v_o^o(n-1) \qquad (4)$$

Substituting (4) into (2’) and taking the Z- Transform we obtain

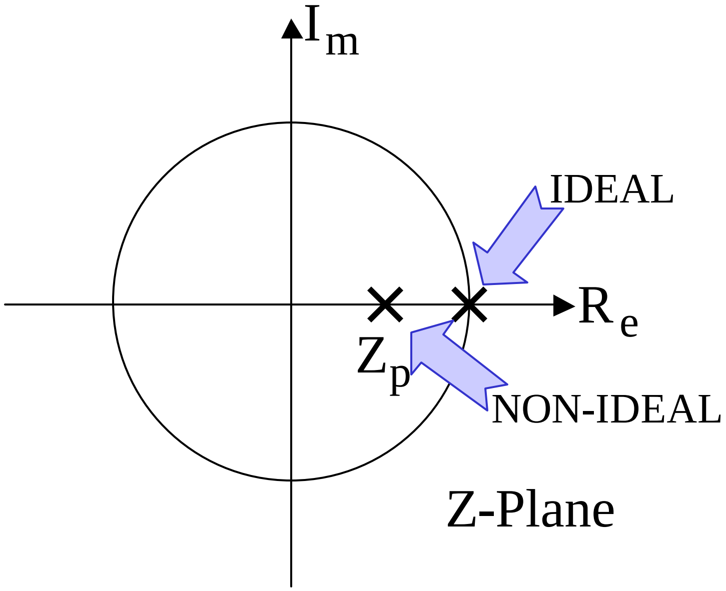
$$-\alpha_1 V_1^o(Z)=\frac{\alpha_1 V_o^o(Z)}{A_o}+\left(1+\frac{1}{A_o}\right)\left(1-Z^{-1}\right)$$

$$H^{oo}(Z)=\frac{V_o^o(Z)}{V_1^o(Z)}=\frac{-\alpha_1}{\frac{\alpha_1}{A_o}+\left(1+\frac{1}{A_o}\right)\left(1-Z^{-1}\right)}=\frac{-\alpha_1}{\frac{\alpha_1}{A_o}+\frac{1}{A_o}+1-Z^{-1}\left(1+\frac{1}{A_o}\right)}$$

$$H^{oo}(Z)=\frac{V_o^o(Z)}{V_1^o(Z)}=\left.\frac{\dfrac{-\alpha_1}{1+\dfrac{1}{A_o}(1+\alpha_1)}}{1-Z^{-1}\dfrac{1+\dfrac{1}{A_o}}{\dfrac{\alpha_1}{A_o}+\dfrac{1}{A_o}+1}}\right|_{A_o\to\infty}=\frac{-\alpha_1}{1-Z^{-1}}$$

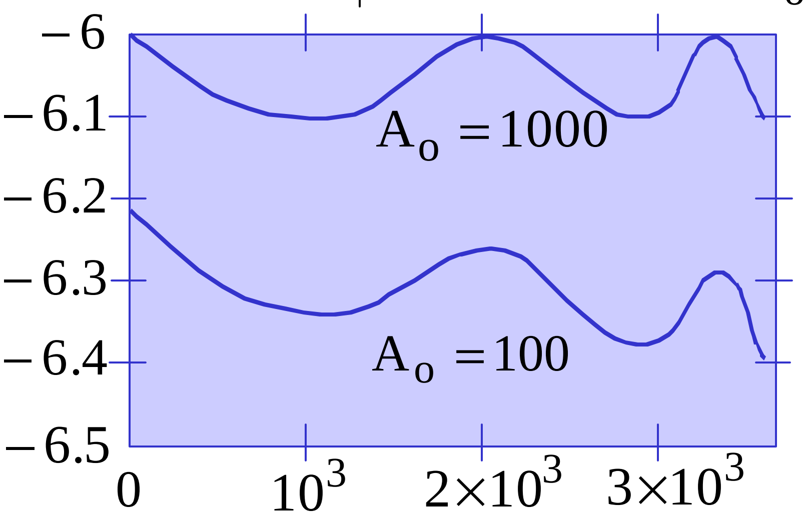
Actual pole located at

$$Z_p = \left. \frac{1+\frac{1}{A_o}}{1+\frac{1}{A_o}(\alpha_1+1)} \right|_{A_o \to \infty} = 1$$



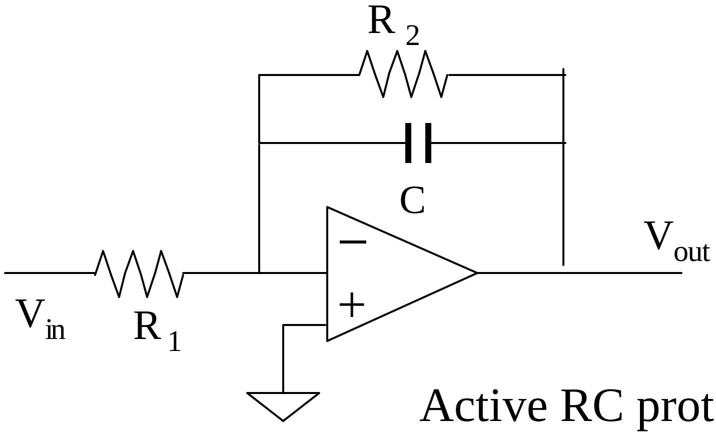
Z-Plane

$$\left|H^{oo}(Z)\right| = \left| \frac{\alpha_1 e^{\omega T/2}}{2j\,\mathrm{sen}(\omega T/2)} \, \frac{1}{1+\frac{1}{A_o}\left(1+\frac{\alpha}{2}\right) - j\frac{\alpha}{A_o \tan(\omega T/2)}} \right|$$



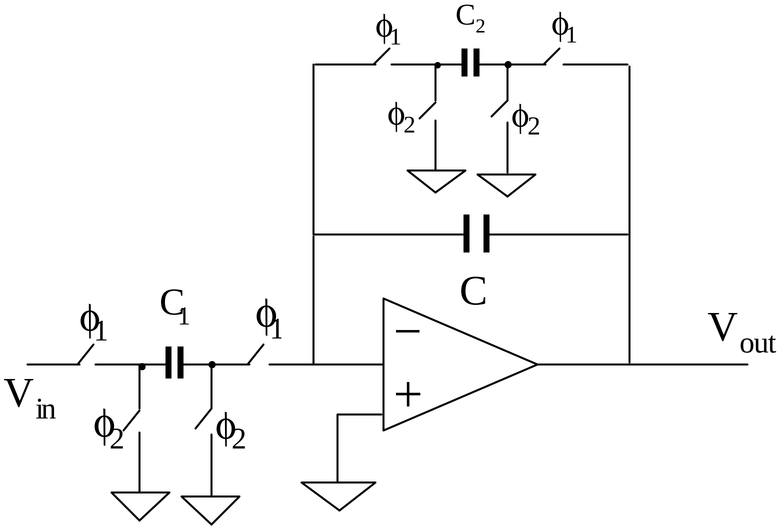
EXAMPLE
5th Order
Chebyshev

Basic SC first-order low-pass



Active RC prototype

$$H(s) = \frac{-R_2/R_1}{1 + sCR_2}$$



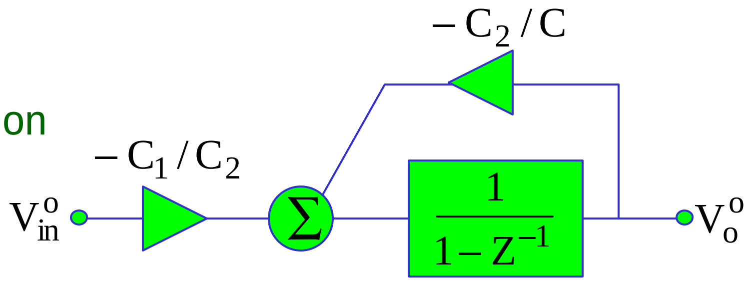
$$R_{eq} = T/C_i = 1/C_i f_S\ ,\ \ i = 1{,}2$$

$$H(z) = \frac{-(C_1/C)z}{z(1 + C_2/C) - 1}$$

$$Z_{zero} = 0$$

$$Z_{pole} = 1/(1 + C_2/C)$$

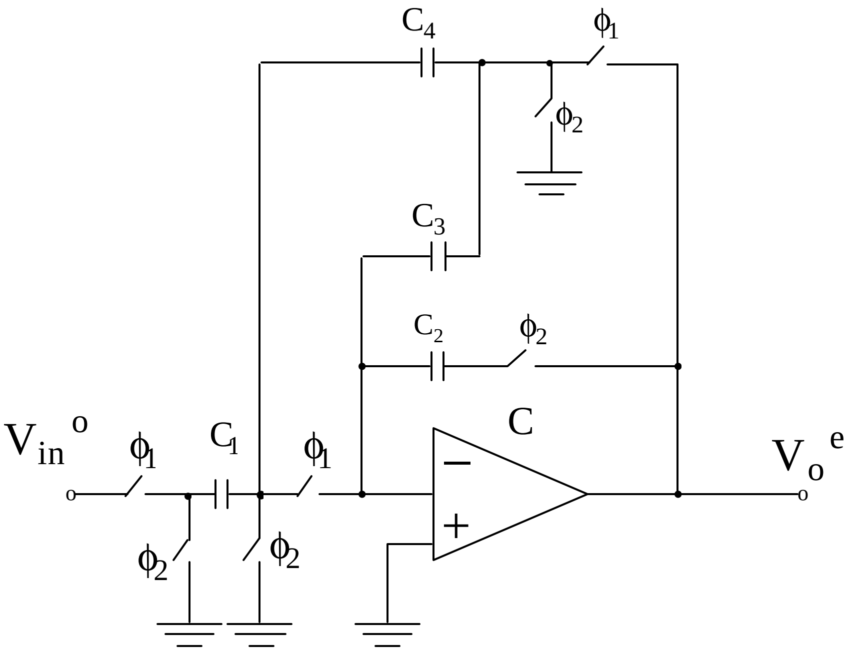
Block Diagram representation



Improved Integrator with reduced capacitance spread

Problem: offset

Solution: use an offset & low dc gain compensated integrator as the 2nd stage (only valid for two-integrator loop applications)

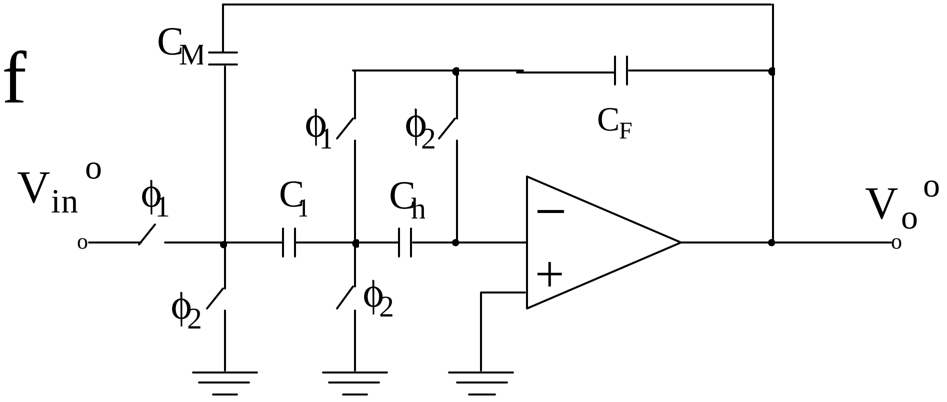


$$H^{oe}(z) = \frac{V_o^e(z)}{V_{in}^o(z)} = -\frac{C_1 C_3}{C_2 C_4'} \frac{1}{1 - z^{-1}}$$

$$C_4' = C_4 + C_3$$

Offset and gain compensated Integrator

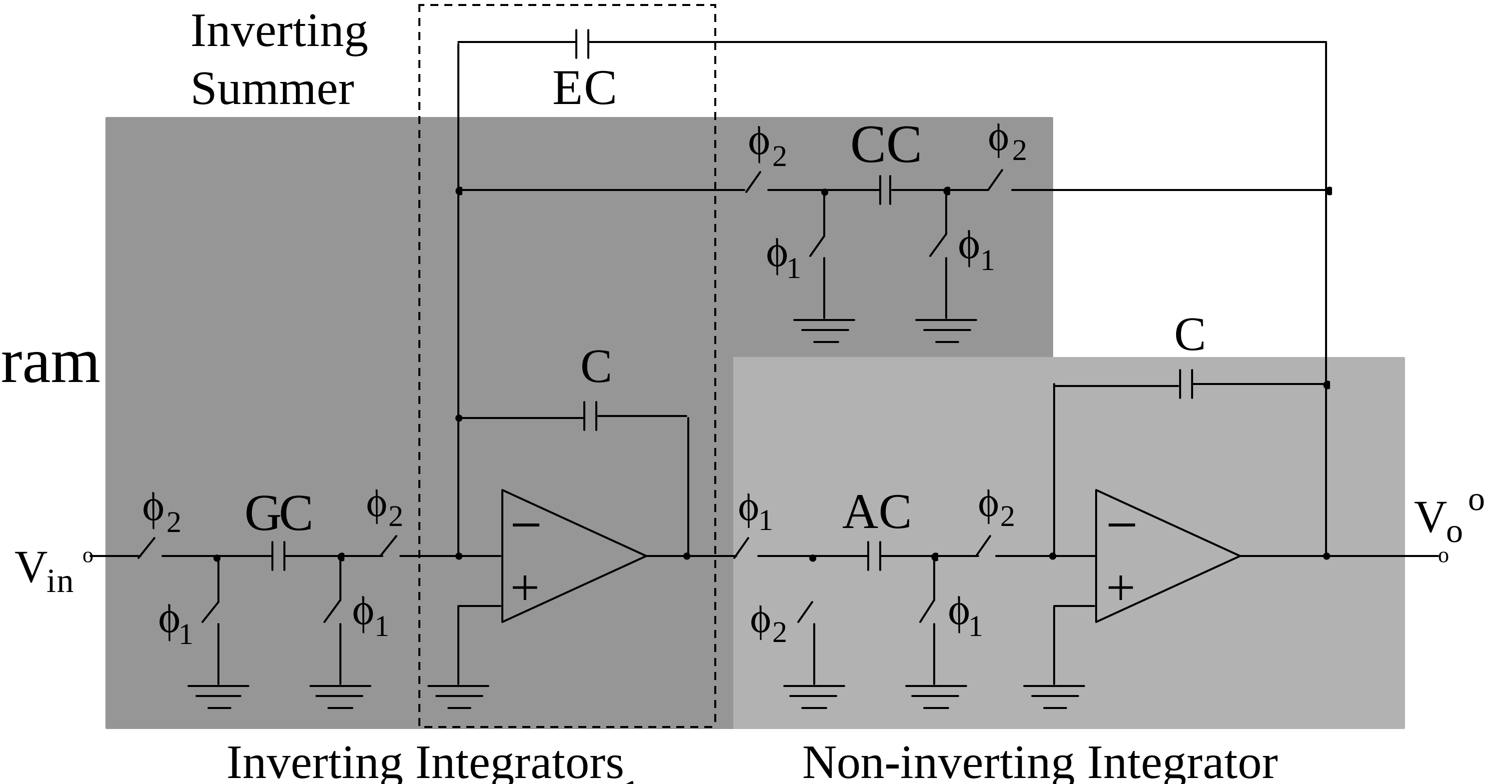
- C_h compensates the offset voltage and dc gain error of the op amp
- C_M eliminates spikes (providing continuous feedback to the op amp)



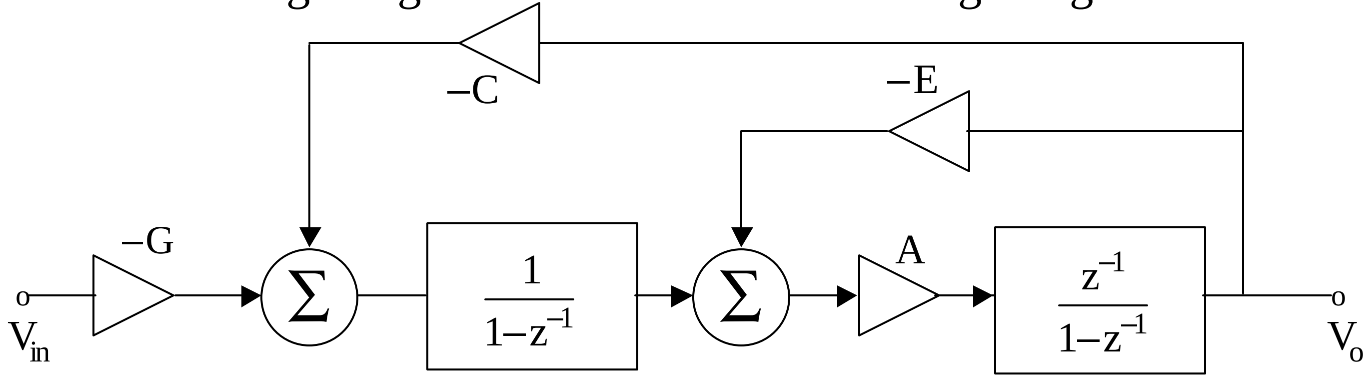
$$H^{oo}(z)=\frac{V_o^{o}(z)}{V_{in}^{o}(z)}=-\frac{C_1}{C_F}\frac{1}{1-z^{-1}}$$

Biquad Circuits & Block Diagrams

Circuit Diagram

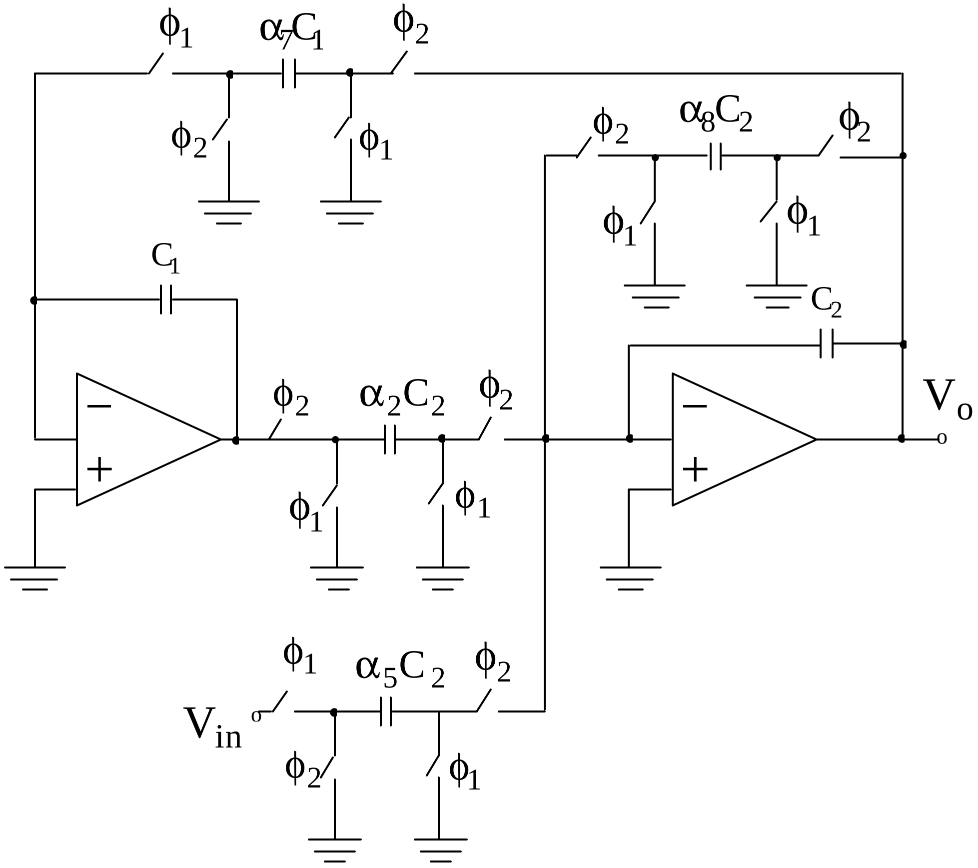


Block Diagram



Biquad Circuit 1

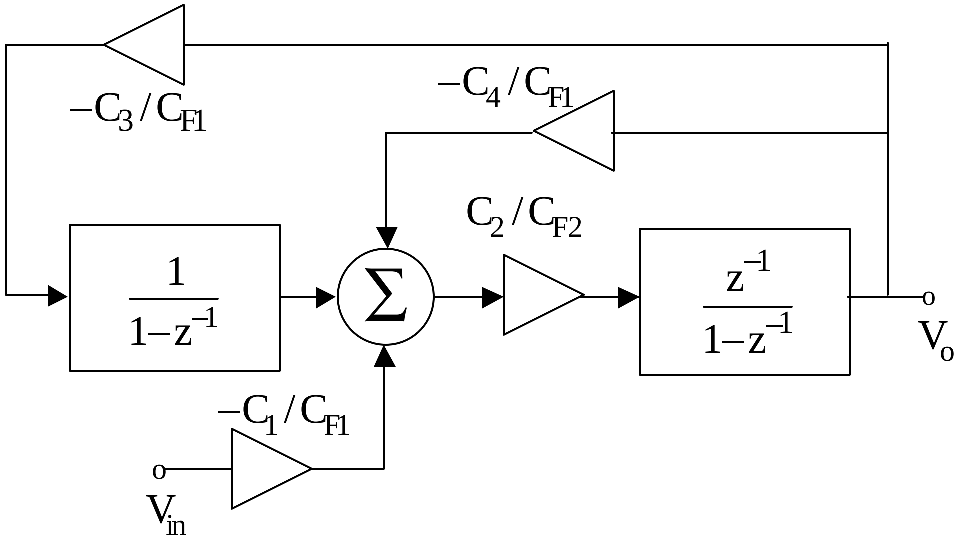
Circuit Diagram



Block Diagram

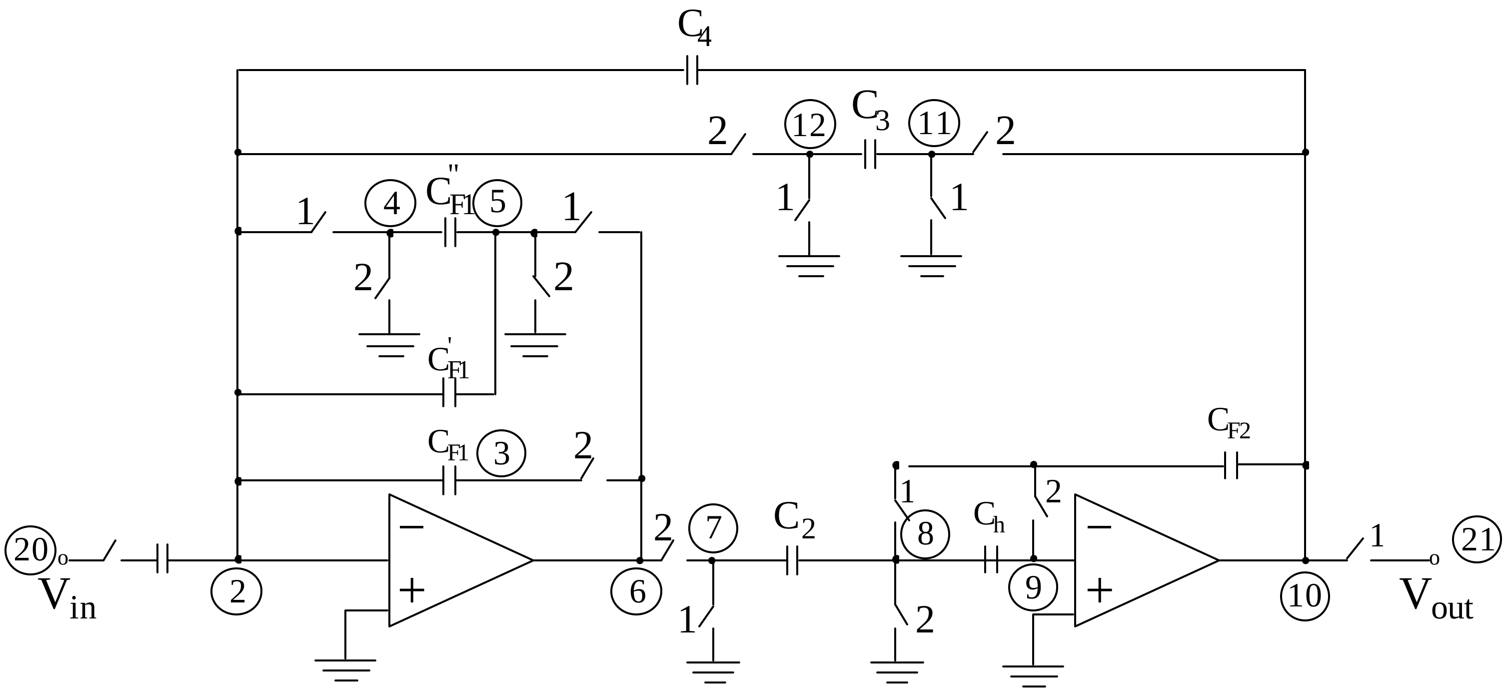
Biquad Circuit 2

Block Diagram

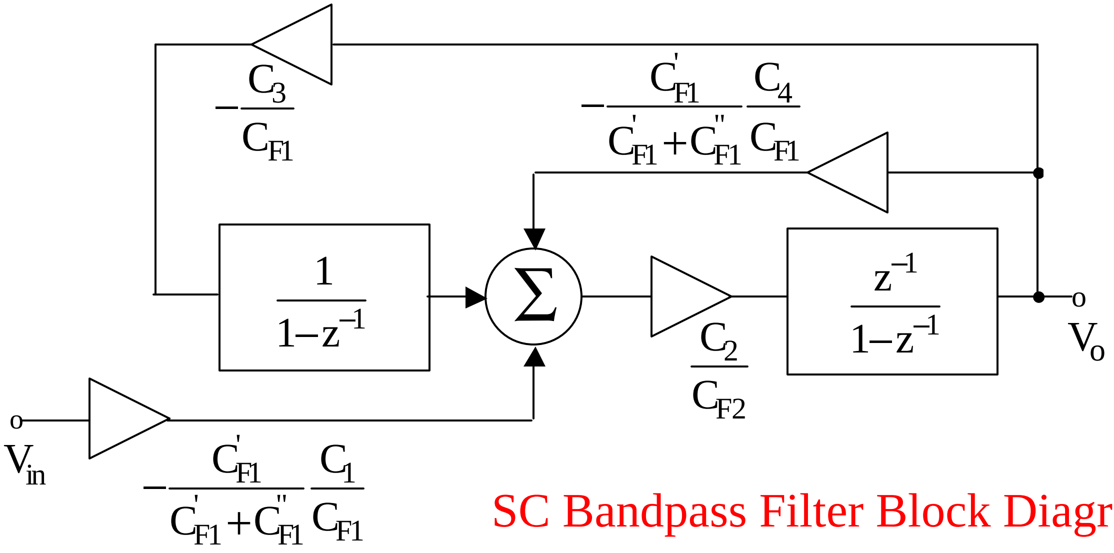


Circuit Diagram

?



SC Bandpass Diagram



SC Bandpass Filter Block Diagram

SWITCAP INPUT FILE

```
timing;
period 2e-5;
clock clk 1 (0
1/2);
clock rq 1/100 (0
1/200);
end;

subckt (1 2 3 4)
opamp (P:a0);
e1 (5 0 3 4) 1;
e2 (1 2 8 0) a0;
s1a (5 6 ) rq;
s1b (8 6) #rq;
s2a (8 7)  rq;
s2b (5 7) #rq;
ceq (6 7) 14.03;
cp (8 0) 10000;
end;

circuit;
c1   ( 1  2) 2.2;
cf1  ( 2  3) 5.4;
cf11 ( 2  5) 1.1;
cf12 ( 4  5) 3;
c2   ( 7  8) 1;
c3   (11 12) 1;
c4   ( 2 10) 1;
cf2  (13 10) 5.4;
ch   ( 8  9) 1;
xe1  ( 6 0 0 2)
opamp (28000);
xe2  (10 0 0 9)
opamp (28000);
s1   (20  1)  clk;
s2   (3  6) #clk;
s3   (5  0) #clk;
s4   (5  6)  clk;
s5   (4  0) #clk;
s6   (2  4)  clk;
s7   (6  7) #clk;
s8   (7  0)  clk;
s9   (8  0) #clk;
s10  (8 13)  clk;
s11  (9 13) #clk;
s12 (21 10)  clk;
s13 (11 10) #clk;
s14 (11  0)  clk;
s15 (12  2) #clk;
s16 (12  0)  clk;
v1 (20 0);
end;

analyze sss;
infreq 1 3000 lin
300;
set v1 ac 1.0 0.0;
print vm(21);
plot vm(21);
end;

end;
```

Capacitor Values for the SC Filter

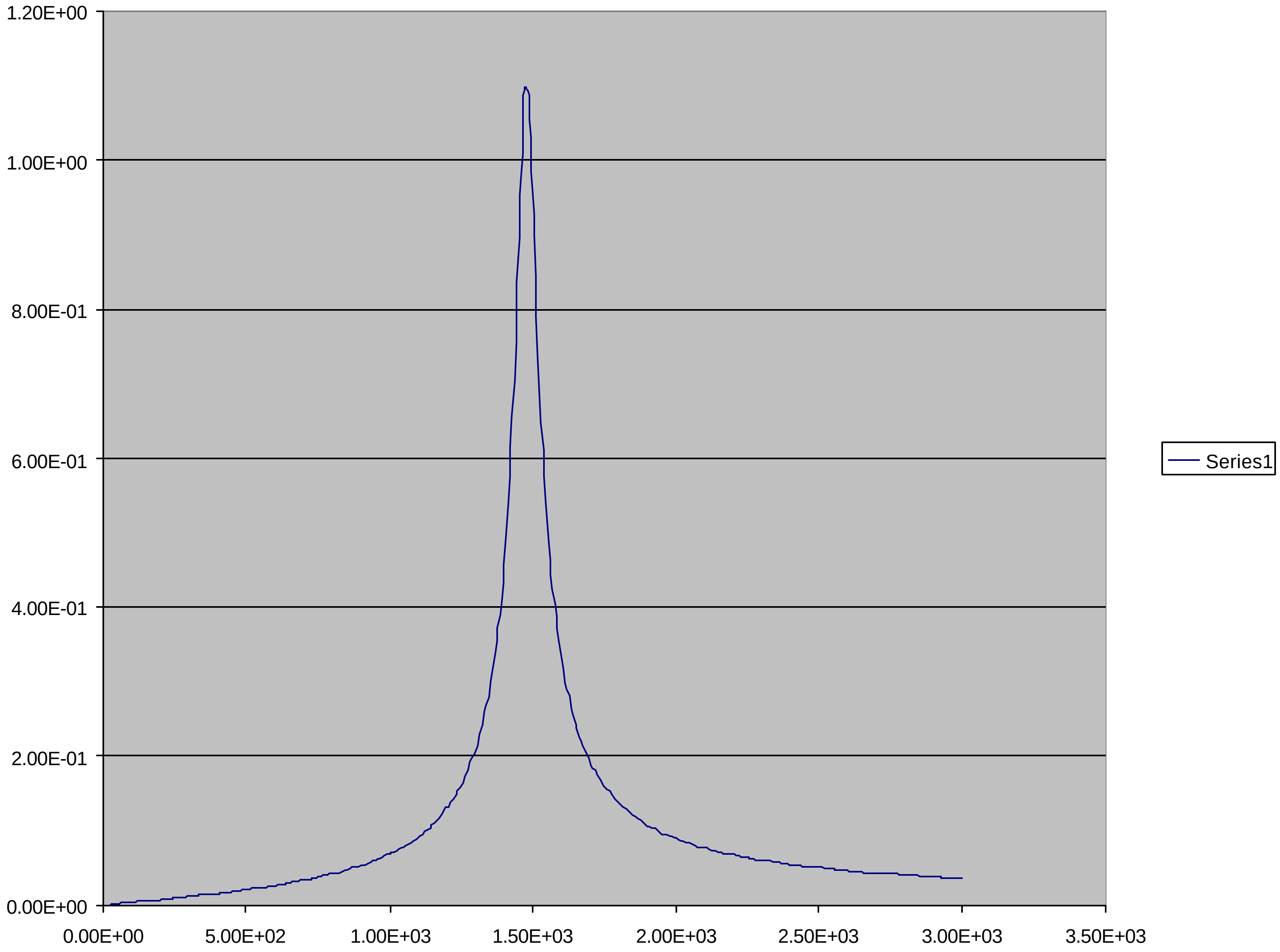
f_o(Hz)	Capacitor Values $C_{F1}^{'}$(Cu)	$C_{F1} = C_{F2}$(Cu)
697	4	11.4
852	2.6	9.3
1209	1.5	6.6
1477	1.1	5.4

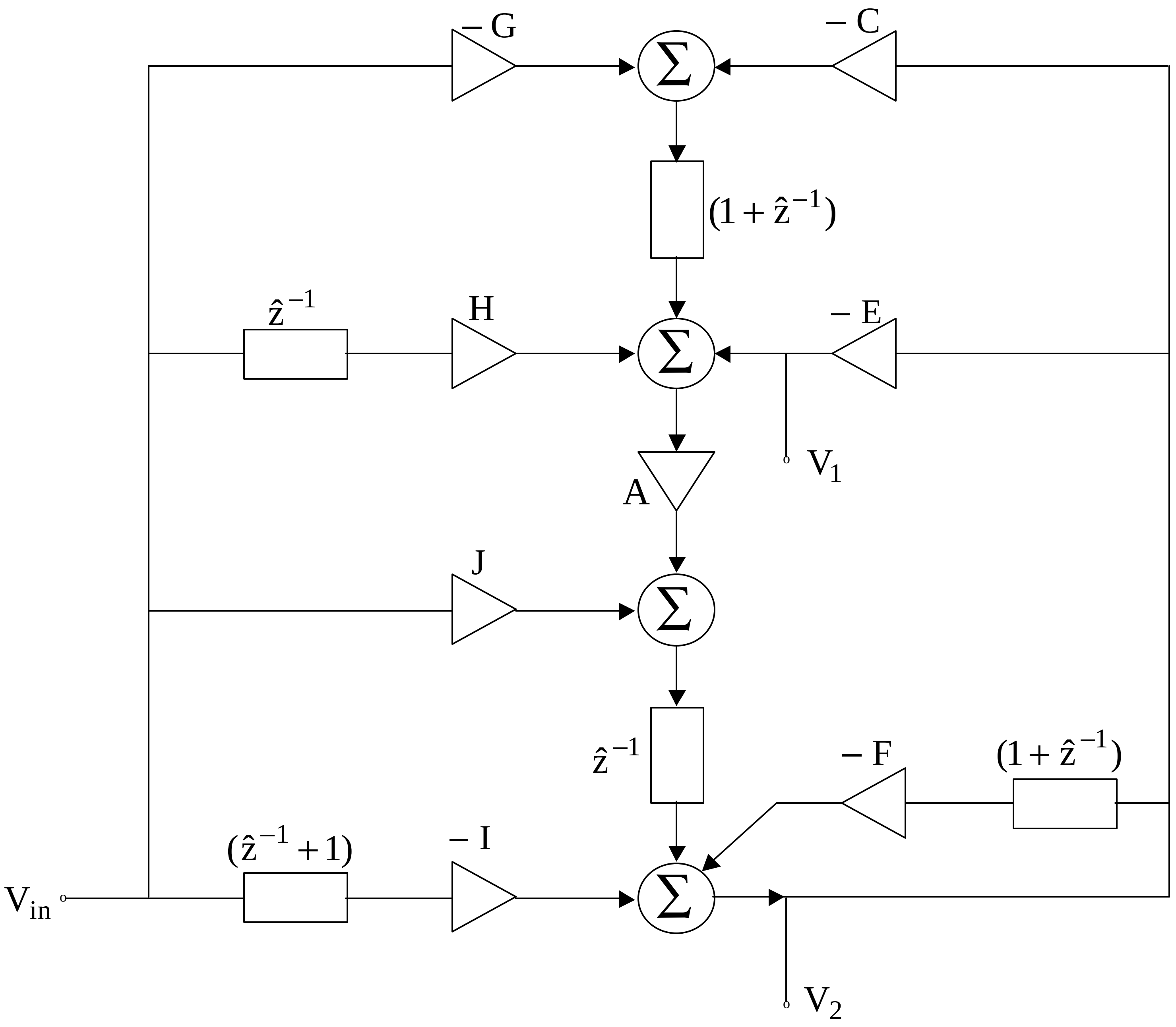
Where: $C_2 = C_3 = C_4 = C_h = 1\,\mathrm{Cu}$, $C_1 = 2.2\,\mathrm{Cu}$,

$C_{F1}^{''} = 3\,\mathrm{Cu}$ and $f_c = 1/T = 50\,\mathrm{kHz}$

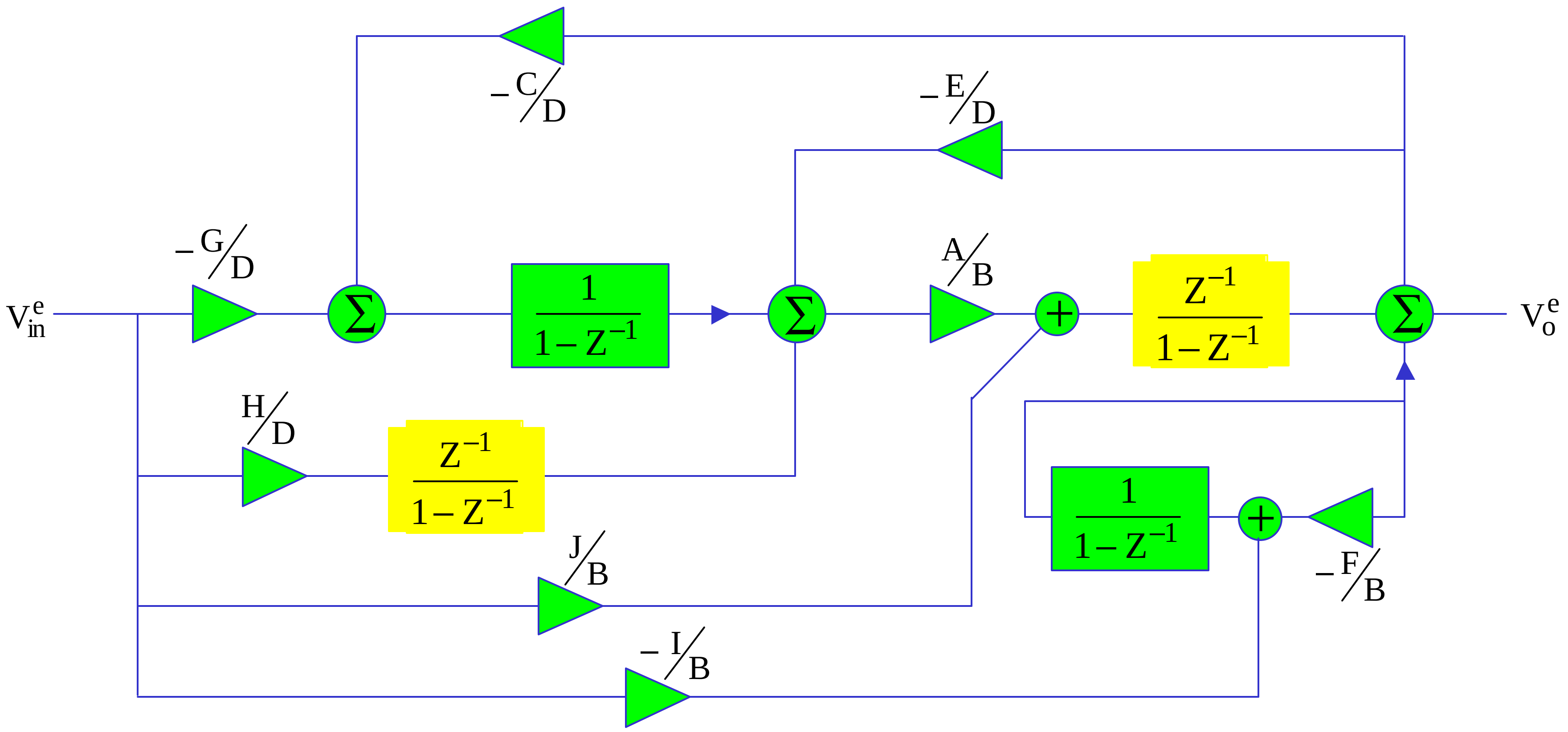
Specifications vs. Simulated Results for the SC Filter

Design Specifications Gain (v/v)	f_o(Hz)	Q	Simulated Results Gain (v/v)	f_o(Hz)	Q
1	697	20	0.998	699.00	19.92
1	852	20	0.999	857.25	20.02
1	1209	20	1.000	1209.00	19.83
1	1477	20	1.002	1479.00	20.11

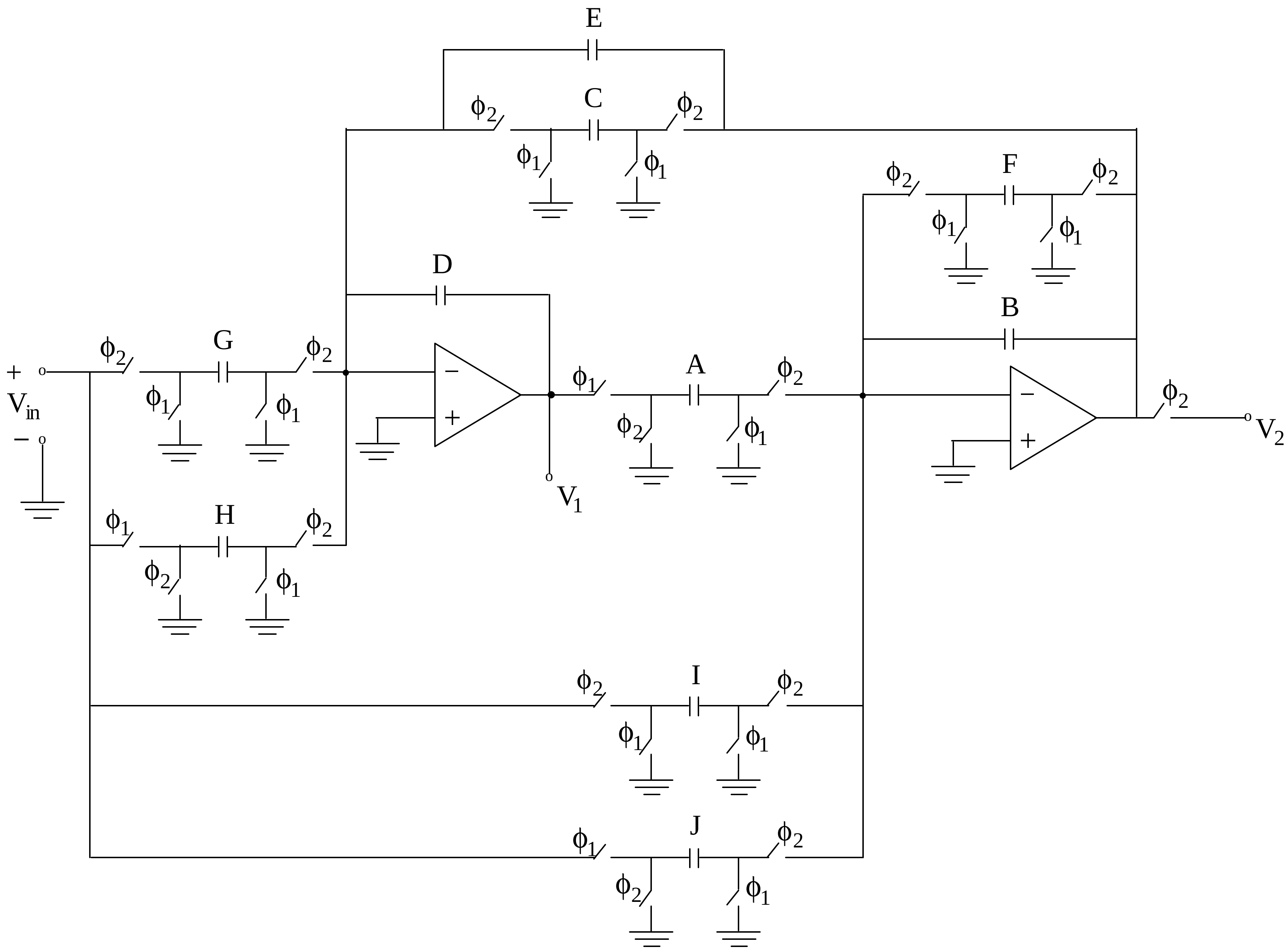

1.20E+00
1.00E+00
8.00E-01
6.00E-01
4.00E-01
2.00E-01
0.00E+00
0.00E+00
5.00E+02
1.00E+03
1.50E+03
2.00E+03
2.50E+03
3.00E+03
3.50E+03
Series1



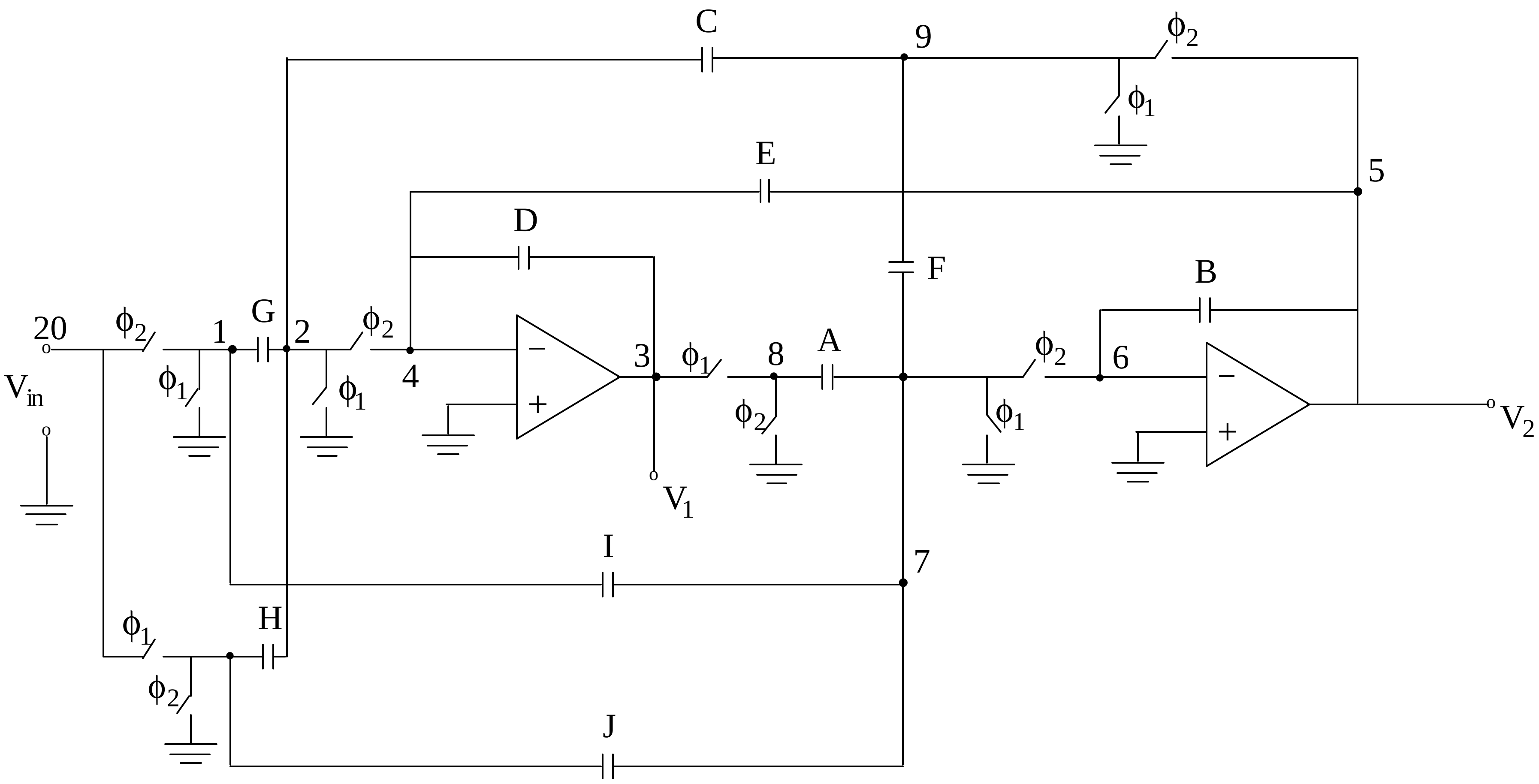
A general SC biquad flow diagram, for B=D=1, type 1. Note that $\hat{z} = z - 1$



An Alternative Block Diagram Representation



SC implementation with maximum number of switches.Type E1 corresponds to F=0



SC implementation minimum switch configuration.

Design of 2nd-Order LP Notch Filter

Design a LP Notch Filter with f_z=1,800 Hz, f_p= 1,700 Hz, Q_p=30 and 0dB DC Gain. The corresponding H(s) yields:

$$H(s) = \frac{0.89195s^2 + (1.140926\times10^8)}{s^2 + 356.0475s + (1.140926\times10^8)}$$

By using the bilinear mapping

$$s = K\frac{1 - z^{-1}}{1 + z^{-1}}$$

where $K = \omega a / \tan(\omega_d T/2)$

For T= 1/128kHz, the corresponding transfer function becomes:

$$H(z) = 0.89093\frac{1 - 1.99220z^{-1} + z^{-2}}{1 - 1.99029z^{-1} + 0.997232z^{-2}}$$

Next we will match the coefficients of H(z), for A=1, with the ones of a SC Biquad Topology, i.e.

$$H_{E_2}(\hat{z}) = -\frac{I + (I + G - J)\hat{z}^{-1} + (G - H)\hat{z}^{-2}}{1 + (E + C)\hat{z}^{-1} + C\hat{z}^{-2}}$$

Output taken at node V_2

where $\hat{z}^{-1} = \dfrac{z^{-1}}{1 - z^{-1}}$; $\hat{z} = z - 1$

$$H(\hat{z}) = 0.89093\frac{1 + 0.0078\hat{z}^{-1} + 0.0078\hat{z}^{-2}}{1 + 0.0097\hat{z}^{-1} + 0.000694\hat{z}^{-2}}$$

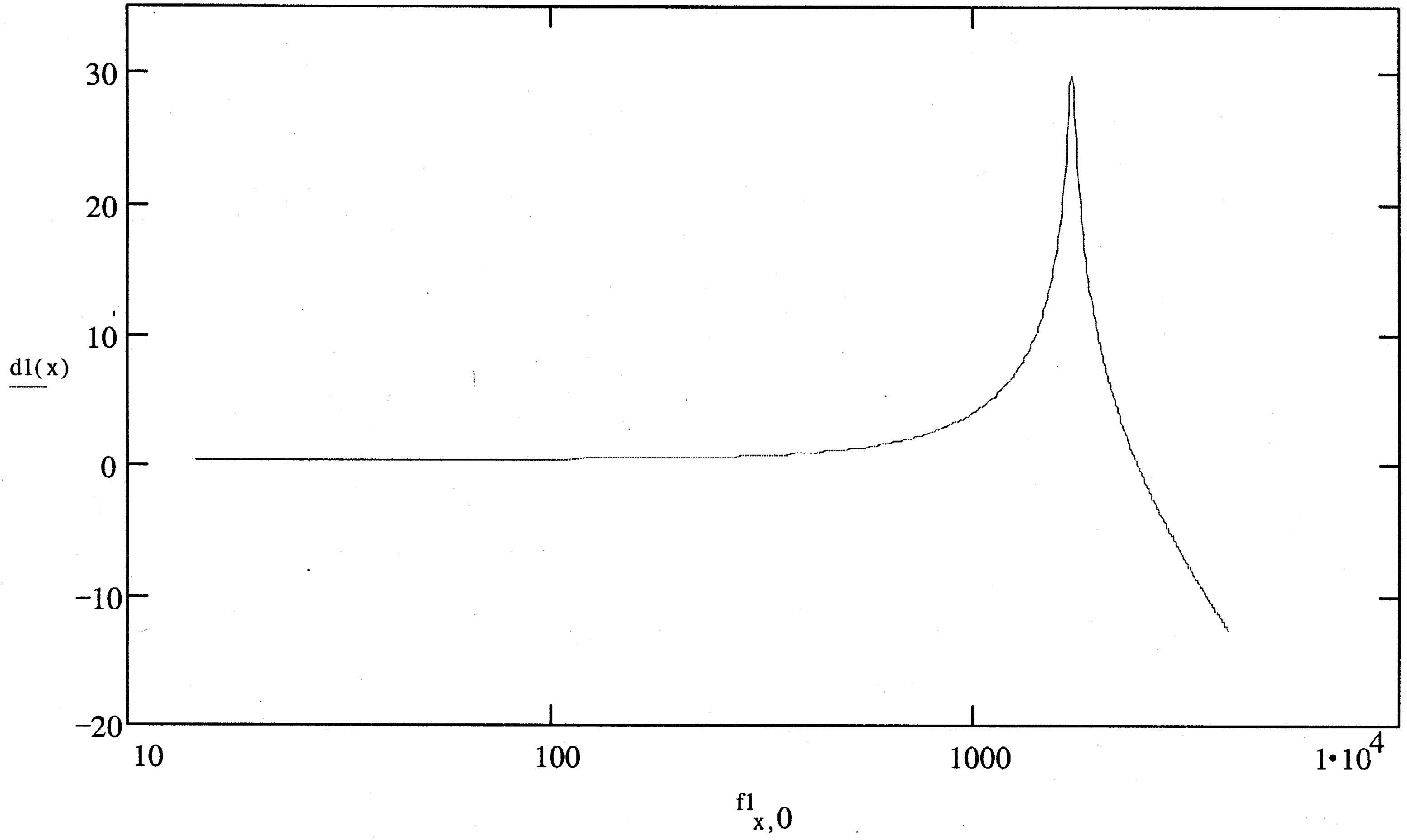
CAPACITOR (PF)	E-CIRCUIT		
	INITIAL	DYNAMIC RANGE ADJUSTED	FINAL
A	1.0000	0.08308	1.0000
B	1.0000	1.0000	12.0365
C	0.00694	0.00694	2.5035
D	1.0000	0.08308	29.9613
E	0.00277	0.00277	1.0000
F	---	---	---
G	0.00694	0.00694	2.5035
H	---	---	---
I	---	---	---
J	---	---	---
K(I=J)	0.89093	0.89093	10.7238
$\sum$ C(pF)	---	---	59.7

SWITCAP Input file

```
timing;
period 7.8125e-06;
clock clk 1 (0 1/2);
end;

circuit;
cg   (1 2) 2.5935;
ca   (8 7) 1.0000;
cd   (4 3) 29.9613;
cb   (6 5) 12.0365;
ce   (4 5) 1.00000;
cc   (2 9) 2.5035;
e1   (3 0 0 4) 28000;
e2   (5 0 0 6) 28000;
s1   (20 1) #clk;
s4   (2 4) #clk;
s5   (9 5) #clk;
s9   (8 0) #clk;
s10  (7 6) #clk;
s2   (1 0) clk;
s3   (2 0) clk;
s6   (9 0) clk;
s7   (3 8) clk;
s8   (7 0) clk;
v1   (20 0);
end;
analyze sss;
infreq 1 4000 lin 300;
set v1 ac 1.0 0.0;
print vm(5);
plot vm(5);
```

Low-pass notch SC

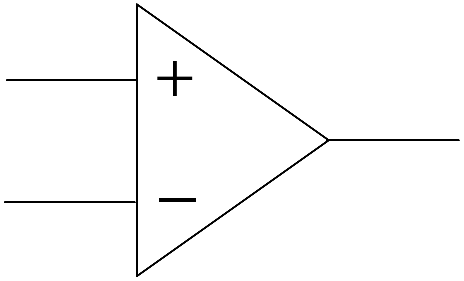


f_p = 1,700 Hz, f_z =1,800 Hz, Qp = 30, DC Gain = 0 dB

PRACTICAL CONSIDERATIONS FOR OP AMPS

Key op amp specifications:

- Gain, output impedance
- Speed (BW)
- Supply voltages
- Output swing
- Noise
- Power



Alternatives ?

NON-IDEAL EFFECTS OF OP AMPS

A. FINITE DC GAIN

For a two integrator biquadratic filter:

$$\omega_{o_A} = \frac{A_o}{1 + A_o}\omega_o \qquad\qquad Q = \frac{1}{\frac{1}{Q} + \frac{2}{A_o}} \cong \left(1 - \frac{2Q}{A_o}\right)Q$$

Therefore:

- ω_o deviations are negligible
- Q deviations can be significant

B. FINITE BANDWIDTH

- Bandwidth is very critical for high frequency applications

Consider A Biquadratic Function

- A_o Effects on w_o and Q

If Q ↑ then A_o↑ High Q requirements imply high A_o

The actual Q_A is given by

$$Q_A = Q\left[\frac{1}{2Q+A_o}\right]\left[1+A_o^2+\frac{A_o}{Q}\right]^{\frac{1}{2}}$$

$$Q_A \approx \left(1-\frac{2Q}{A_o}\right)Q$$

and the actual center frequency w_A is

$$\omega_A = \omega_O\left[\frac{1}{1+A_o}\right]\left[A_o^2+\frac{A_o}{Q}+1\right]^{\frac{1}{2}} \approx \omega_O \frac{A_o}{A_o+1}$$

In fact in a LP function:

$$\omega_{peak} = \omega_o\left[1 - \frac{1}{2Q^2}\right]^{\frac{1}{2}}$$

and the change of magnitude of denominator $|D|$ is

$$\Delta|D| = 20\log\frac{Q}{Q_A} \cong 20\log(1 + 2Q/A_o)$$

Examples

$A_o = 10^3$

i) Q = 1 then

$w_{peak} = 0.707\ w_A$

$Q_A = 0.998$

ii) Q = 10

$w_{peak} = 0.997\ w_A$

$Q_A = 1.00125Q = 9.8$

iii) Q = 15

$\Delta|D| \cong 0.257\text{dB}$

$Q_A \cong 14.56$

How to determine the GB of an Op Amp?

- The required GB is a function of the clock frequency and the feedback topology around the Op Amp.

- A rule of thumb to select GB requires to satisfy the following inequality:

$$a \, GB \, T > 5$$

where T is the period of the clock frequency, a is the capacitor ratio between the sum of all the feedback capacitors divided by the sum of all the capacitors connected to the input terminal of the Op Amp

Architectures Styles and Pros and Cons

A. SINGLE-ENDED CONFIGURATION

- High gain
- CMRR = 0
- Nested compensation trades BW for stability

B. FULLY DIFFERENTIAL CONFIGURATION

- High gain
- Good CMRR
- Rail-to-rail output swing
- Higher bandwidth (less compensation)
- Requires and additional CMFB and dynamic reset

C. FULLY BALANCED FULLY DIFFERENTIAL

- Use single ended amplifiers
- Excellent CMRR

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